

RUSSIAN ECONOMIC AND SOCIAL HISTORY

SIMPLE EQUATIONS OF THE FEL'DMAN MODEL

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In Fel'dman's model there are two sectors, sector 1 produces the means of production for both sectors, sector 2 produces consumption goods. Initial capital stock is divided into each sector, k^p and k^c , and the problem is the choice of investment. Capital cannot shift between sectors. In each sector the marginal capital output ratio ($\frac{dK}{dY}$) is given, and constant.¹ Let $y_t^p = ak_t^p$ be output in the producer good sector, and $y_t^c = bk_t^c$ be output in the consumer goods sector (by assumption a and b are both constant). Labor is supplied elastically to both sectors.

The key decision is how much of the production goods should be invested in each sector. Let α be the share invested in production goods. Thus we have $i_t^p = \alpha y_t^p$ and $i_t^c = (1 - \alpha)y_t^p$. Let δ be the constant rate at which capital depreciates. Therefore:

$$k_t^p = (1 - \delta)k_{t-1}^p + i_t^p \quad (1)$$

and

$$k_t^c = (1 - \delta)k_{t-1}^c + i_t^c \quad (2)$$

We can write equation (1) as

$$k_t^p = (1 - \delta)k_{t-1}^p + \alpha y_t^p = (1 - \delta)k_{t-1}^p + \alpha(ak_t^p) \quad (3)$$

or

¹The sectoral assumptions may make some sense in early development where the differences are great, and intersectoral linkages are weak.

$$k_t^p = \frac{(1 - \delta)}{1 - \alpha a} k_{t-1}^p \quad (4)$$

and for the consumer goods sector, using equation (2)

$$k_t^c = (1 - \delta)k_{t-1}^c + (1 - \alpha)y_t^p = (1 - \delta)k_{t-1}^c + (1 - \alpha)(ak_t^p) \quad (5)$$

or

$$k_t^c = \frac{(1 - \delta)(1 - \alpha)}{1 - \alpha a} k_{t-1}^p \quad (6)$$

Since output in the consumer goods sector is just a linear function of the capital stock in the consumption goods sector, we have:

$$y_t^c = b \left[\frac{(1 - \delta)(1 - \alpha)}{1 - \alpha a} \right] k_{t-1}^p \quad (7)$$

You can see from (7) that the output of the consumer goods sector depends on the capital stock of the production goods sector. Although raising α directly reduces the amount of investment devoted to consumer goods, it indirectly increases it, since it makes the capital stock of the production sector grow faster. The question is how long must we wait for this effect.

Notice that equation (7) is a simple difference equation. Set $a = .25$, $b = .33$, and $\delta = 0$, then we can try different values for α and see how consumption behaves, as in figure 1:

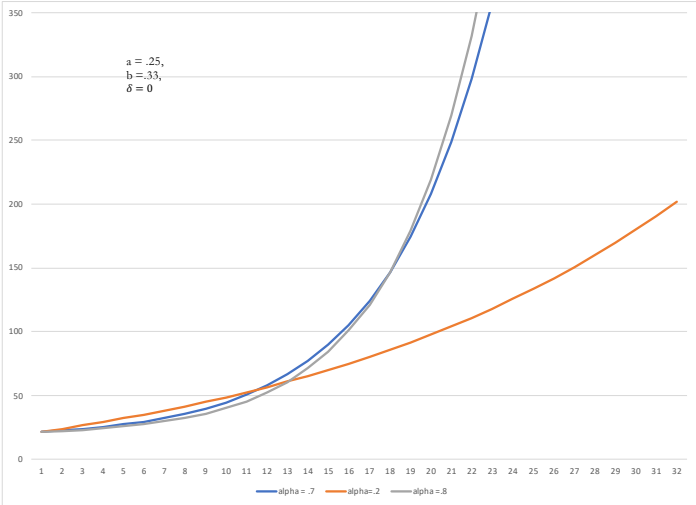


FIGURE 1. Impact of Investment Choice on Consumption in the Fel'dman Model