Econ 497 Professor Ickes

Economics of the Financial Crisis Spring 2012

Homework Assignment #1: Answer Key

1. Consider a firm that has future payoff, X. Suppose the firm is unlevered, call the firm U, and its shares S_U. Now suppose that an identical firm (L) with the same future payoff issues debt (D_L) and denote its shares S_L. Consider two portfolios: one that consists of $\alpha\%$ of the shares of U, and the second that consists of $\alpha\%$ of the bonds of L plus $\alpha\%$ of the shares of L. What are the total current costs of these two portfolios equal to? What are the payoffs of these two portfolios equal to? If investors can engage in short selling what would we expect the cost of these two portfolios to be equal to? Explain. What have you shown in fact?

   brief answer *The current cost of the unlevered portfolio is $\alpha S_U = \alpha V_U$, and the payoff is αX. For the levered firm we have $\alpha D_L + \alpha S_L = \alpha V_L$, with payoff $\alpha \min(X, rD_L)$ for the debt, and $\alpha \max(0, X - rD_L)$ for the equity. To see why, note that the debt receives rD_L if there is sufficient payoff, but if not, they receive whatever is leftover – the equity holders are residual claimants. Hence, they receive what is left over after the debt is paid off. So the payoff of this portfolio is*

$$\begin{aligned} & \alpha \min(X, rD_L) + \alpha \max(0, X - rD_L) \\ = \; & \alpha[\min(X, rD_L) + \max(0, X - rD_L)] \\ = \; & \alpha X \end{aligned}$$

   so the payoffs are the same. Hence, we have the Modigliani-Miller theorem.

2. Suppose a firm generates $\$100M$ in profits for sure every period in perpetuity, and this is the only cash flow it has. Suppose the risk-free interest rate, $r = .10$. Assume that the firm faces a corporate income tax, with a tax rate, $t_c = .40$.

   (a) If the firm is equity financed what is the value of the firm? What would it be if $t_c = 0$?

   brief answer *The value of the firm is the present value of the after-tax cash flow. Each period the after-tax cash flow is $FCF = (1 - t_c)100 = (1 - .40)100 = 60$. So $V = \frac{FCF}{r}\frac{60}{.10} = \$600M$. If the tax rate is zero, then $FCF = (1 - 0)100 = 100$, $V = \frac{100}{.10} = 1000$.*

   (b) Now suppose that the firm takes on $\$500M$ of debt, and that this debt is risk free. How much do the debt holders get each period?

   brief answer *Debt is risk free and $r = .10$, so the coupon is $.10(500M) = 50M$.*

   (c) What happens to the value of the firm? Explain.

   brief answer The equity value depends on the income after the bondholders are paid. Net income of the firm is now $(1 - .4)(100 - 50) = 30$. Value of equity is then $\frac{30}{.10} = 300$. The value of the firms is $V = E + D = 300 + 500 = 800$. So the firm's value has increased by \$200.

Net income of the firm is now $(1-.4)(100-50)=30$. *Value of equity is* $\frac{30}{.10}=300$. *Value of the firms is* $V=E+D=300+500=800$. *So the firm's value has increased by \$200. The firm is saving* $.40(50)=20$ *each year in taxes, and* $\frac{20}{.10}=200$ *is thus the PV of the tax shield. Who gains from the debt? Suppose that there were initially 1000 shares outstanding, so before the debt was issued each share was worth* $\frac{600}{1000}=.60$. *Now the firm uses 500 to buy back shares. Think of the firm as now consisting of* 500 *in cash plus its value as an unlevered firm, plus the PV of its tax shield, or*

$$500+600+200=1300.$$

(d) Suppose that the firm uses the proceeds of the debt to buy back $\$500M$ worth of shares. What happens to the market value of shares?

brief answer *The value of the equity* $E=1300-500=800$, *so the price per share (recall there are 1000 shares) has increased from* .60 *to* .80. *Equity holders get the whole gain of the tax shield. The firm buys* $\frac{500}{.80}=625$ *shares back with the proceeds of the debt issue. So the value of the firm is now* $1300-500=800$. *So* $E=V-D=800-500=300$. *Notice that the firm's market capitalization has fallen but the equity holders wealth is now higher,* $=300+500=800$.

(e) Given your analysis, under these conditions what is the optimal level of debt for the firm? Are you surprised?

brief answer *Under these assumptions the optimal debt/equity ratio is100 percent. The tax shield is independent of the level of debt, if the debt is risk free. All excess cash should be paid to shareholders. The corporate income tax is providing incentives for the firm to increase its debt levels.*

3. This problem considers the implications of financial distress. Consider a firm that has assets in place which pay off next period depending on the state of nature, which can be either *boom* or *bust.* Assume that the probability of each state is 0.5. Let the payoff to the firm in the boom be 100, and the payoff in the bust be 20. Assume everyone is risk neural, $r=0$, and there are no taxes. So the value of the firm, $V=60$. Assume the firm has debt with face value, $F=50$.

(a) What is the value of the equity and debt [hint: calculate the payoffs for debt and equity in the boom and bust and take expected values]? How does the value of debt compare with its face value?

brief answer *The payoffs for debt and equity are given by:* $D=\min(V,F)$, $E=\max(V-F,0)$. *So*

$$\begin{array}{ll} \textit{boom} & D=F=50, E=50 \\ \textit{bust} & D=V=20, E=0 \end{array}$$

The values today depend on the expected values, and since the states occur with equal probability, we must have: $D=35, E=25$. *The value of the firm is* $D+E=60$. *This means that the debt is under water, it is trading below par, since* $D=35<50=F$.

(b) Now suppose that the firm has a new investment that will create profits for sure. If we invest 10 today, we get 15 tomorrow in either state of the world. What happens to the

firm's cash flow and values in both states if it takes on the investment? What happens to the firm's value?*Now the payoffs in each state are given by*

$$\begin{array}{ll} \textit{boom} & \text{cash flow equals } 100 + 15 = 115 \\ \textit{bust} & \text{cash flow equals } 20 + 15 = 35 \end{array}$$

so the firm's value increases from 60 *to* 75.

(c) Will equity holders invest the 10 today in this investment?

brief answer *No. Even though* $V = 75$*, the issue is how is the value split between equity and debt. We now have:*

$$\begin{array}{ll} \textit{boom} & D = 50, E = 65 \\ \textit{bust} & D = 35, E = 0 \end{array}$$

so today the value of $D = 42.5$ *and* $E = 32.5$*. Both have increased by* 7.5*, but equity will not invest* 10*, since they would lose* 2.5 *in value. There is a wealth transfer from the equity holders to the debt holders if they invest. Debt is senior and gets part of the surplus. There is a debt overhang that prevents a profitable investment from taking place. Notice this is arising because we have risky debt, whose value is affected by the investment. If we had risk-free debt it would not happen.*

(d) Can we finance the new investment with debt?

brief answer *Not if it is junior. Then the same problem happens. Part of the surplus is transferred to the old debt. If we could issue new debt that was senior – risk free – then it is no problem. In this case:*

$$\begin{array}{lll} \textit{boom} & \text{cash flows } = 115. \text{ New } D = 10, & \text{old } D = 50, E = 55 \\ \textit{bust} & \text{cash flows } = 35. \text{ New} D = 10, & \text{old } D = 25, E = 0 \end{array}$$

so $E = 27.5$ *and old* $D = 37.5$*, everybody is better off. But this does not happen. Why? Typically, there are covenants that prevent the firm from issuing senior debt. This is insisted on by bondholders to prevent the value of their debt from being reduced in value.*

(e) What do your results suggest about the costs of too-much debt financing?

brief answer *It suggests that there are costs of too much debt. We lose the chance to take actions that would be beneficial because of the debt overhang. This is a bankruptcy-related cost. It implies that we may not want to choose the debt levels implied by problem 2.*

4. In the PSU-Michigan football game, the possible outcomes are as follows. In the first half, PSU can go ahead by 10 points or Michigan can go ahead by 10 points. In the second half, the change in the score can be 10 points in PSU's favor or 10 points in Michigan's favor. PSU has the better team, so in the first half there is a 70% probability that PSU will take the lead. However PSU's team gets too conservative when ahead, so if it ends the first half in the lead, there is a 50% probability that the score will move in Michigan's favor in the second half. If Michigan's team ends the first half in the lead, the probability remains 70% that the score will move in PSU's favor in the second half.

(a) Draw an event tree that describes the outcomes in this game. [Hint: in figure 1 I start the event tree for the first half of the game. At each node of the tree, enter the PSU lead (the number of points that PSU is ahead, a negative number if Michigan is ahead). What are the probabilities of each final PSU lead (i.e., of each final score, PSU's lead can be positive or negative)?

brief answer *The event tree is given below. There are three final outcomes: (i) PSU up by* 20 *has a probability of* $70\% \times 50\% = 35\%$*; (ii) a tie has a probability of* $70\% \times 50\% + 30\% \times 70\% = 56\%$*; (iii) PSU loses by* 20 *has a probability of* $30\% \times 30\% = 9\%$.

(b) Suppose there are bookmakers who allow you to bet at the start of the game on the halftime score. If the bookmakers have no costs and make no profits on average, how much do you have to pay at the start of the game to receive \$1 if PSU is ahead at half-time? How much do you have to pay to receive \$1 if Michigan is ahead?

brief answer *Since the bookmakers make no profits, whatever they take in has to be what they expect to pay out. For someone buying a "PSU Ahead" bet,* $P = 70\% \times \$1 + 30\% \times \$0 = \$0.70$*, i.e., you pay the expected value. Similarly, for someone buying a "Michigan Ahead" bet,* $P = 70\% \times \$0 + 30\% \times \$1 = \$0.30$.

(c) Suppose the bookmakers also operate at half-time, and allow you to bet on the final score. If PSU is ahead at half-time, how much do you have to pay at half-time to receive \$1 if PSU wins the game, and how much do you have to pay at half-time to receive \$1 if the game is a tie? What if Michigan is ahead at half-time?

brief answer *The logic is similar to part b). If PSU is ahead (conditional on...), "PSU Wins" bets are* \$0.50 *and "PSU Ties" bets are* \$0.50*. If PSU is behind (conditional on...), "PSU Ties" bets are* \$0.70 *and "PSU Loses" bets are* \$0.30.

(d) Show that even if the bookmakers do not offer bets at the start of the game on the final outcome of the game, you can still use a sequence of bets, starting at the beginning of the game, to obtain \$1 if PSU wins the game, or \$1 if Michigan wins, or \$1 if the game is a tie. How much does it cost to obtain each of these payments?

brief answer *Note that what you are buying is a riskless asset—an asset that pays \$1 no matter what the outcome is. Let's look at this going backwards for intuition. If PSU wins the game and we get a dollar, we must have bought a "PSU Wins" ticket, which costs \$0.50. But to do that, we must have had \$0.50 at the "PSU Ahead" point at halftime. To have that, we must have bet \$0.35 on "PSU Ahead" at the start of the game. If PSU loses the game and we get a dollar, we must have bought a "PSU Loses" ticket, which costs \$0.30. But to do that, we must have had \$0.30 at the "PSU Behind" point at halftime. To have that, we must have bet \$0.09 at the start on "PSU Behind." If PSU ties the game and we get a dollar, one of two things happened. Either we bet "PSU Ties" after "PSU Ahead" or we bet "PSU Ties" after "PSU Behind." The first one costs \$0.50, requiring \$0.35 of a bet on "PSU Ahead" at the start. The second bet costs \$0.70, requiring a bet of \$0.21 at the start on "PSU Behind."*

Result *Bet \$0.70 on "PSU Ahead" at the start and \$0.30 on "PSU Behind" at the start. This costs you one dollar. Then no matter what, you end up with one dollar*

at halftime (look at part b). If PSU is ahead, bet \$0.50 on both "PSU Wins" and "PSU Ties," using up your dollar. If PSU is behind, bet \$0.70 on the tie and \$0.30 on the loss, using up your dollar. Then, as we calculated in part c), you end up with a dollar no matter what, because whatever the outcome is, you bought the bet that pays off one dollar.

Conclusion *So it costs you one dollar to get a riskless one dollar in the future. It also is unnecessary for bookies to offer direct bets on the end of the game (note if they did, you'd pay \$0.35 for "PSU Wins," \$0.09 for "PSU Loses," and \$0.56 for "PSU Ties," as we calculated in part a)). By using a sequence of bets, you can construct in effect a bet on the final outcome. Note the price of the bet sequence on any outcome is the expected value of that outcome, because the bookies are not trying to make any money.*

(e) How do these costs relate to the probabilities in part a)? Under what conditions would you expect this relationship to hold in other more general examples?

brief answer *The costs are the probabilities. What if the bookies were trying to make a profit or recoup costs? Then they'd have to "charge" more, which means you'd pay a little more for each bet. You couldn't pay a dollar in initial bets and then get a dollar back in the end as you are now. Another consideration is the riskless rate of interest. As we know from intermediate economics, investment in a riskless asset returns the riskless rate of interest. Here we invest in a riskless asset for the duration of the game. If the risk-free rate of interest is positive, we'd expect that investing a dollar at the start of the game would return slightly more than a dollar by the end of it. In other sorts of bets people make, such as in the financial markets, transaction costs and interest rates will affect the costs of assets.*

Comment *What is neat here is that we have three outcomes here—PSU wins, ties, or loses. But we only have two assets, really—a bet that PSU moves up or falls behind. At first glance, it might seem markets cannot be complete. If they are not complete, we cannot construct a riskless asset (convince yourself of this: A riskless asset is the sum of all Arrow-Debreu securities. If you don't have complete markets, you don't have all Arrow-Debreu securities). But we did construct a riskless asset, because markets are complete. The fact that this is a multiperiod setup allows one more factor - time - to give us the tools to complete markets. There are (at most) only two branches leaving each node on the tree above, so we need (at most) only two assets to complete markets.*

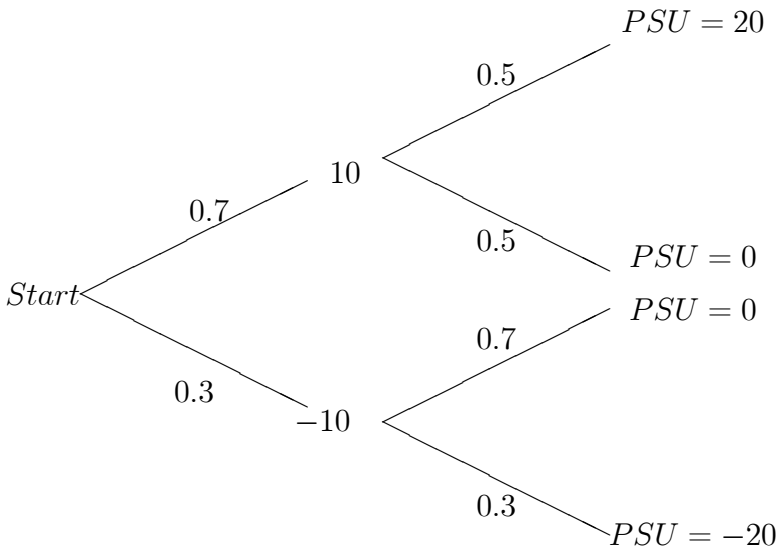

Figure 1: Event Tree Penn State – Michigan