

Homework Assignment #2: Answer Sheet

1. Consider a world with two assets, one is a safe asset in variable supply, the other a risky asset in fixed supply. There are two periods. The payoffs are given by:

	$t = 0$	$t = 1$	
Safe Asset	1	1.5	
Risky Asset	P	6 with prob .25 1 with prob .75	(1)

where P is the price that the asset will cost in period 0 and is to be determined, and suppose all investors are risk neutral. Suppose investors have one unit of wealth and invest their own money. In this case investors should equate marginal returns [I don't think you need a calculator for this question, but it could make your life a bit easier].

- (a) What is the expected value of the risky asset in period 0? The marginal return on the safe asset is $\frac{1.5}{1}$. Given that marginal returns should be equal for the two assets (recall risk neutral investors), what should P be? Call this P_F , the fundamental value of the risky asset.

brief answer *The expected value of the risky asset is given by: $EV = .25(6) + .75(1) = 2.25$, so the marginal return is $\frac{2.25}{P_F}$, where P_F is the unknown price of the risky asset that we are solving for. Equating marginal returns thus implies that $\frac{2.25}{P_F} = \frac{1.5}{1} \implies P_F = \frac{2.25}{1.5} = 1.5$. Notice that we can do this because agents are risk neutral. If agents were risk averse the return on the risky asset would have to be higher than 1.5 so $P_F < 1.5$.*

- (b) Now suppose that investors have no wealth of their own. They can borrow at date 0 and repay 1.33 in date 1 if they are able to. Further suppose that the lenders cannot observe how loans are used. What is the return to investing in the safe asset? Can P_F still equal 1.5 if investors are using borrowed funds?

brief answer *The bank can never make you pay more than 0 so the return on the risky asset is higher with borrowed money since you may not pay back the loan. The return on the safe asset is now given by $R_{safe} = 1.5 - 1.33 = 0.17$. If we borrow one unit and purchase the risky asset we get $\frac{1}{P}$ units, so if the price is unchanged, $P = 1.5$ then we borrow one unit to get $\frac{1}{1.5}$ units of the risky asset. Then*

$$\begin{aligned}
 R_{risky} &= .25\left[\frac{1}{1.5}(6) - 1.33\right] + .75\left[\frac{1}{1.5}(1) - 1.33\right] \\
 &= .25\left[\frac{1}{1.5}(6) - 1.33\right] + .75(0) = .67
 \end{aligned}$$

but $.67 > .17$, so 1.5 cannot be the equilibrium price of the risky asset. Its value must rise.

- (c) What will happen to P given that the supply of this risky asset is fixed? What will P be?

brief answer *Since the return on the risky asset is higher than the fixed asset the price of the risky asset must rise. We know that in equilibrium marginal returns should be equal (again agents are risk neutral by assumption). So we have to solve for P . We have: $.25[\frac{1}{P}(6) - 1.33] + .75(0) = 1.5 - 1.33$, and we solve this for P to get the equilibrium price. So*

$$\begin{aligned} .17 &= .25\left(\frac{6}{P}\right) - .25(1.33) \\ .17 + .25(1.33) &= .25\frac{6}{P} = \frac{1.5}{P} \\ P &= \frac{1.5}{.17 + .25(1.33)} = 3 \end{aligned}$$

Notice that the equilibrium price has risen compared with the case where investors used their own wealth.

- (d) Given the new equilibrium price P that you calculated, how do you interpret this price? Why is the price of the risky asset higher when investors borrow than when they invest their own wealth? Explain.

brief answer *Since $P > P_F$, we could consider this to be a bubble. The price is higher than what we considered it to be fundamentals in part a. Why does this occur? Because investors do not bear all the downside risk with lending. Hence, the return is higher and they demand more of it, and given the fixed supply of the risky asset the price must rise. Lending shifts the risk from the investors to the depositors, and increases the demand for the risky asset driving the price above fundamental values. There is an agency problem here that causes excessive borrowing and drives up the price of the risky asset.*

- (e) Suppose we changed the probabilities and payoffs to make the riskier asset riskier. Consider

	$t = 0$	$t = 1$
Safe Asset	1	1.5
Risky Asset	P	13.5 with prob .1 1 with prob .9

What happens to the fundamental price when there is no lending in this case? What happens to the ratio of $\frac{P}{P_F}$ if there is lending? How does this ratio compare with your calculation in part d?

brief answer *I chose the new probabilities and payoffs so that the expected value would be unchanged, notice that $.1(13.5) + .9(1) = 2.25$, as in part a, so $P_F = \frac{2.25}{1.5} = 1.5$, just like before. But the risky asset is now riskier. The bad state is much more likely than before. Given this fact, with lending, and equating marginal returns, we now have*

$$.17 = .1\left(\frac{13.5}{P}\right) - .1(1.33)$$

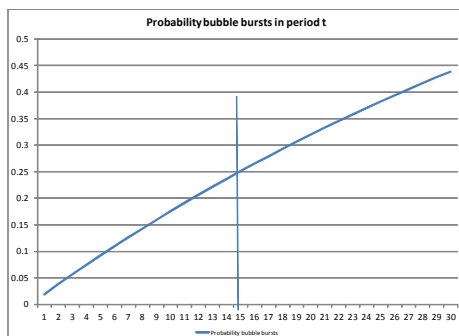


Figure 1:

$$.17 + .133 = \frac{1.35}{P}$$

$$P = \frac{1.35}{.17 + .133} = \frac{1.35}{.305} = 4.426$$

so $\frac{P}{P_F} = \frac{4.426}{1.5} \approx 2.96$. In part c we had $\frac{P}{P_F} = \frac{3}{1.5} = 2$, so the bubble has increased in size.

2. Suppose the fundamental price of an asset is constant and equal to 100 and that the interest rate is constant and equal to 3%. Suppose further that the actual price of the asset is growing 5% per period. Can this be a rational bubble? [for this question a calculator or a program like excel may make your life easier]

- (a) Can you calculate the probability that the bubble will continue for another period? Call this q and calculate it. How many periods would you expect the bubble to persist before the likelihood that it bursts equals .25? Explain.

brief answer Since the bubble is growing faster than the rate of interest it could be a rational bubble. If the bubble could burst, we know that $b_{t+1} = \frac{(1+r)b_t}{q}$, if the bubble has not yet burst. So $q = \frac{(1+r)b_t}{b_{t+1}}$, and $b_{t+1} = (1.05)b_t$, so $q = \frac{(1+r)b_t}{b_t(1.05)} = \frac{1.03}{1.05} = 0.98095$. The probability that the bubble bursts in period 1 is then $1 - q = 1 - .9809 = 0.0191$. The probability that the bubble lasts for two periods is q^2 , so the probability it bursts by period 2 is $1 - q^2$, and thus the probability it bursts for sure in n periods is $1 - q^n = 1 - .9809^n$, and if $n = 15$ we have $1 - .9809^{15} = 0.25119$. It is rather easy to just put this in excel. If we plot $1 - q^n$ we get:

- (b) Suppose that the interest rate is 5% and that q is the same number you calculated in part a. At what rate must the bubble price grow for this to be a rational bubble? Explain.

brief answer We still know that $b_{t+1} = \frac{(1+r)b_t}{q}$, but now we have $b_{t+1} = \frac{1.05b_t}{q}$ or $.9809 = \frac{(1+r)b_t}{b_t(1+x)} = \frac{1.05}{1+x}$, so

$$.9809(1+x) = 1.05$$

$$1+x = \frac{1.05}{.9809}$$

$$x = \frac{1.05}{.9809} - 1 = .07044$$

so the bubble must now grow faster at 7.04%. Given that the bubble still collapses with the same probability as before, we need faster growth to compensate for the risk that the bubble collapses.

- (c) Suppose the interest rate was 4%. What happens to the number of periods that you calculated it would take before the likelihood that the bubble bursts reaches .25?

brief answer We start with $q = \frac{(1+r)b_t}{b_t(1.05)} = \frac{1.04}{1.05} = 0.99048$. This is less than in part (a) because the interest rate is now higher, so the bubble is actually smaller. Continuing as before, the probability that the bubble bursts in period 1 is then $1-q = 1-.999048 = .000952$. The probability it bursts for sure in n periods is $1 - q^n = 1 - .999048^n$, and if $n = 27$ we have $1 - .999048^{27} = .25388$. Since the probability it bursts any period is lower it takes longer now.

3. Suppose that dividends grow at some constant rate, g , and that the return on equity is some constant rate, k . Let D_0 be the current level of dividends. What should the price of a share of stock be? Assume that $k > g$, then the following useful fact will help you: if $x > y$ then $\sum_{j=1}^{\infty} \left(\frac{1+y}{1+x}\right)^j = \frac{1+y}{x-y}$. [for this question a calculator or a program like excel may make your life easier]

brief answer The price of a stock should equal the present discounted value of the dividend stream. So we should have $p_t = \sum_{j=1}^{\infty} \frac{D_{t+j}}{(1+k)^j}$, but since dividends grow constant at rate g , we have $p_t = \sum_{j=1}^{\infty} \frac{D_t(1+g)^j}{(1+k)^j} = \sum_{j=1}^{\infty} D_t \left(\frac{1+g}{1+k}\right)^j$, and given the fact I gave you, we have $p_t = D_t \frac{1+g}{k-g}$. This is called the Gordon growth model. It gives an estimated of the stock price as a function of future earnings, if we can associate the expected dividends with this constant growth rate.

- (a) Given your expression for the price of a stock, suppose that $D_0 = \$20m$, and that $k = 9.2\%$ and dividends grow at 8% . What should P_0 be? Suppose that dividends are now expected to grow at 7.5% .

brief answer Now we just plug into our expression, $p = \$20m \frac{1.08}{.092-.08} = \$1,800m$. If dividend growth is instead 7.5% , we have $\$20m \frac{1.075}{.092-.075} = \$1264.7m$.

- (b) What happens to the value of the stock? Does this seem like a small or large change given the magnitude of the change in dividend growth?

brief answer We get a fall in the value of shares of approximately 30%, even though the fall in dividend growth is almost imperceptible. Notice, however, that while the change from 8% to 7.5% seems very small in terms of an expected change in the growth rate of future dividends, this is a permanent change in the growth rate. It is expected to persist forever. That is why the effect on current price is so large.

- (c) Suppose that g was unchanged but that stocks became riskier so k increased to 9.9% . What happens to the price of the stock now?

brief answer We now have $p = 20 \frac{1.08}{.099-.08} = \$1136.8M$. Again a very large fall in the stock price from a very small change in perceptions of the future.

- (d) How does this relate to bubbles, if at all?

brief answer *This tells us to be cautious about what we call bubbles. A very small change in the perception about the future can cause a very large change in the "proper" price of an asset, if these changes are expected to be permanent.*