Econ 434 Professor Ickes

Fall 2006

## Homework Assignment #1: Answer Sheet

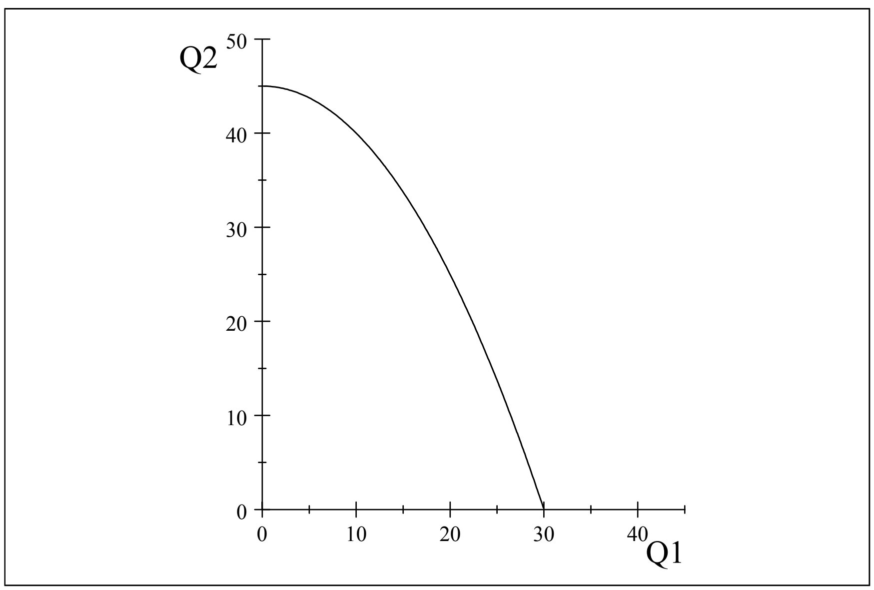
*This assignment is due on Tuesday, Sept 19, at the beginning of class (or sooner) .*

1. *Consider a small open economy that is endowed with* 30 *units of the consumption good in period one and* 0 *in period two. This is a one-good economy, but we also have investment. Specifically, period one output can be transformed into period two output according to the production frontier,*

$$Q_2 = 45 - .05Q_1^2. \tag{1}$$

*Draw a two-period diagram with this endowment point and the production frontier suggested by (1) (hint: if none of the endowment is consumed in period one (fat chance of that!) what is $Q_2$?). Does this production frontier exhibit diminishing returns to investment? Explain.*

**brief answer** See the figure below. Yes, it has diminishing returns. Slope of the production frontier is $-.1Q_1$, which clearly gets smaller as $Q_1$ gets smaller.




Production Possibilities

(a) *Suppose that the world interest rate is* 10%*. How would you find the optimal production point?*

**brief answer** Where the present value of production is maximized. That is where the line with slope $-(1+r)$ is equal to the slope of the production frontier. Anywhere else has lower present value. This is evident in figure 1

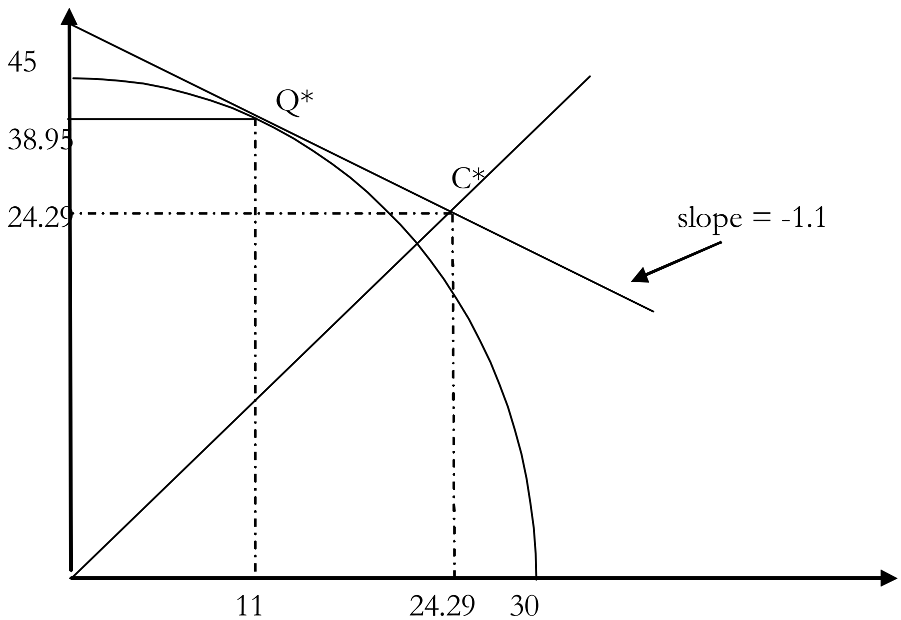



Figure 1: Optimal Production and Consumption Decision

(b) *What is the optimal production point [hint: the slope of the production frontier is given by the first derivative of expression (1) with respect to $Q_1$]?*[1] *If the interest rate fell to zero what would happen to the production point?*

**brief answer** Set $-(1+r) = -1.1$ to slope of the production frontier, $-.1Q_1$:

$$\begin{aligned} -1.1 &= -.1Q_1 \\ Q_1 &= 11 \\ Q_2 &= 45 - .05(11)^2 = 38.95 \end{aligned}$$

If the interest rate fell to zero then $-(1+0) = -.1Q_1 \Rightarrow Q_1 = 10, Q_2 = 40$. Production moves to the northwest along the frontier. This makes sense. A lower interest rate means that future production is even more profitable. So you produce more second period goods.

(c) *Suppose that for this country preferences are such that $\beta = \frac{1}{1+.1}$. Calculate the optimal consumption bundle [hint: recall the intertemporal budget constraint]. Will this country run a current account surplus or deficit in period one? Can you calculate its size? What is the value of the current account balance in period two*?

**brief answer** If $\beta = \frac{1}{1+r}$ then optimal consumption is equal in both periods, $C_1 = C_2$. The intertemporal budget constraint tells us that

$$C_1 + \frac{C_2}{1+r} = Q_1 + \frac{Q_2}{1+r}$$

[1] *Suppose you have a function $y = 1 - x^\alpha$. The first derivative of $y$ with respect to $x$ is $-\alpha x^{\alpha-1}$. The derivative is the slope of the function evaluated at that point. It gives the change in variable, $y$, when $x$ changes, for infinitesimally small changes in $x$.*

but $C_1 + \frac{C_2}{1+r} = C_1 + \frac{C_1}{1+r} = C_1\left(1 + \frac{1}{1+r}\right)$, so

$$C_1\left(1 + \frac{1}{1+r}\right) = C_1\left(1 + \frac{1}{1+.1}\right) \approx C_1(1.91)$$

$$\begin{aligned} 1.91C_1 &= 11 + \frac{38.95}{1.1} \\ 1.91C_1 &= 46.40909 \\ C_1 &= C_2 = \frac{46.40909}{1.91} = 24.29 \end{aligned}$$

Since $C_1 > Q_1$ the country has a current account deficit in period one, $= 24.29 - 11 = 13.29$ In period two the current account surplus is $38.95 - 24.29 = 14.66$. Notice that the present value of the current account in period two $\frac{14.66}{1+.1} = 13.3$, so the present value of the current account balances over the two periods $= 0$, as we expect.

2. Suppose that domestic investment and savings are given by:

$$\begin{aligned} I_{US} &= 100 - 3r_{us} \\ S_{US} &= 20 + 6r_{us} \end{aligned}$$

nd that investment and savings in the rest of the world are given by:

$$\begin{aligned} I_{ROW} &= 120 - 7r_{\text{row}} \\ S_{ROW} &= 40 + 6r_{\text{row}} \end{aligned}$$

(a) *Suppose the US is a closed economy. What will the interest rate be in the US? What will the interest rate be in the rest of the world?*

**brief answer** In a closed economy $I_{US} = S_{US}$, so $20 + 6r_{us} = 100 - 3r_{us}$. Thus,

$$\begin{aligned} 9r_{US} &= 80 \\ r_{US} &= \frac{80}{9} = 8.889 \end{aligned}$$

In the ROW we have

$$\begin{aligned} 120 - 7r_{\text{row}} &= 40 + 6r_{\text{row}} \\ 80 &= 13r_{\text{row}} \Longrightarrow r_{\text{row}} = \frac{80}{13} = 6.154 \end{aligned}$$

(b) *Suppose that the US opens up to the rest of the world. What will the world interest rate be equal to?*

**brief answer** Market-clearing now requires that the world interest rate satisfy:

$$I_{US} + I_{\text{row}} = S_{US} + S_{\text{row}}$$

or

$$\begin{aligned}
100 - 3r^* + 120 - 7r^* &= 20 + 6r^* + 40 + 6r^* \\
100 + 120 - 20 - 40 &= r^*(6 + 6 + 3 + 7) \\
160 &= 22r^* \\
r^* &= 160/22 = 7.2727
\end{aligned}$$

We can note that $r^*$ is less than the US autarky rate but above the ROW autarky rate $(6.154 < 7.2727 < 8.889)$ , which we know must be the case.

(c) *At the equilibrium world interest rate calculate net savings in the US and the ROW. Will the US have positive net savings?*

**brief answer** We know that at $r^*$ the US will have negative net savings. This follows because at $r_{US} = 8.889$ we had $I_{US} = S_{US}$, and $r^* < 8.889$, so savings will decrease and investment will be higher. So we know the answer. But we are supposed to calculate it. We can substitute into the savings and investment functions to obtain the answer. For the US, we have:

$$\begin{aligned}
I_{US} &= 100 - 3(7.2727) = 78.182 \\
S_{US} &= 20 + 6(7.2727) = 63.636
\end{aligned}$$

so the $CA_{US} = 63.636 - 78.182 = -14.546 < 0$. For ROW we have:

$$\begin{aligned}
I_{ROW} &= 120 - 7(7.2727) = 69.0911 \\
S_{ROW} &= 40 + 6(7.2727) = 83.636
\end{aligned}$$

so $CA_{ROW} = 83.636 - 69.0911 = 14.545$. Fortunately for us, these sum to zero (subject to rounding error). That tells us we have the right answer.

(d) *Suppose that US savings increases. Specifically, suppose it shifts upwards by* 20 *at any* $r$. *What happens to the autarky interest rate in the US? What happens to the equilibrium world interest rate if the US is open?*

**brief answer** The autarky rate must fall as this creates an excess of savings over investment in the US at the old autarky rate. We see that now we have $40 + 6r_{us} = 100 - 3r_{us} \Longrightarrow 9r_{us} = 60 \Longrightarrow r_{us} = \frac{60}{9} = 6.6667$, which is, indeed, less than 8.889. If capital markets were liberalized then the world interest rate will be lower. As the US autarky rate is now 6.667, we know that $r^*$ must be between 6.154 and 6.667.

(e) *Calculate net savings in the US and the ROW at the new equilibrium world interest rate. Will the US have positive net savings?*

**brief answer** Follow the same procedure as in parts (b) and (c) but with the new US savings function:

$$\begin{aligned}
100 - 3r^* + 120 - 7r^* &= 40 + 6r^* + 40 + 6r^* \\
100 + 120 - 40 - 40 &= r^*(6 + 6 + 3 + 7) \\
140 &= 22r^* \\
r^* &= 140/22 = 6.3636
\end{aligned}$$

which does satisfy our last assertion in part (d). Now just use this value as we did in part (c):

$$\begin{aligned} I_{US} &= 100 - 3(6.3636) = 80.909 \\ S_{US} &= 40 + 6(6.3636) = 78.182 \end{aligned}$$

so $CA_{US} = 78.182 - 80.909 = -2.727$, so the US has a very small current account deficit. You can guess what the answer is for ROW. But why not just calculate to make sure.

$$\begin{aligned} I_{ROW} &= 120 - 7(6.3636) = 75.455 \\ S_{ROW} &= 40 + 6(6.3636) = 78.182 \end{aligned}$$

so $CA_{\text{row}} = 2.727$, which is satisfying, since $-2.727 + 2.727 = 0$.

3. Consider a two-country world with one time period. There is one good, but income is stochastic. There are two states of nature, denoted 1 and 2. The probability of each state is given by $\pi_1 = \pi_2 = 0.5$. The identical agents in each country maximize expected utility, given by

$$\begin{aligned} EU &= \pi_1 c_1 + \pi_2 c_2 - \frac{1}{2}\left[(c_1 - \overline{c})^2 + (c_2 - \overline{c})^2\right] \\ &= \pi_1 c_1 + \pi_2 c_2 - \frac{1}{2}\sum_i (c_i - \overline{c})^2 \end{aligned}$$

where $c_i$ $(i = 1, 2)$ is the state that actually occurs, and $\overline{c} = \frac{c_1+c_2}{2}$ is the average level of consumption. In state 1 income in country $A$ $(y_1^A)$ is 100 and in state 2, $y_2^A = 50$. In country $B$, $y_1^B = 25$ and $y_2^B = 75$.

(a) *Suppose that there is no trade across countries. Calculate expected utility in each country.*

**brief answer** If autarky then we have $c_i = y_i$ in each state for each country.

$$EU^A = 0.5(100) + 0.5(50) - 0.5\left[(100 - 75)^2 + (50 - 75)^2\right] = -550$$

$$EU^B = 0.5(25) + 0.5(75) - 0.5\left[(25 - 50)^2 + (75 - 50)^2\right] = -575$$

(b) *Suppose the two countries can trade an asset that has a state-contingent payoff. The price of the asset is unity. Can these two economies trade across states to increase their expected utility? Explain. What would the contract look like (who trades how much in each state)?*

**brief answer** Obviously country $A$ would like to offer some of the good in state 1 to compensate for the risk of state 2. For country $B$ it is just the opposite. Suppose that country $A$ trades 25 to country $B$ in state 1 in exchange for the receipt of 25 in state 2. This is mutually advantageous, and given that each state is equally likely has great benefits. See figure 2 Under autarky we are at point $E_0$. Moving to the northwest utility rises for both countries. At point $E_1$ the gains from trade are exhausted. Indeed, for country $A$ average consumption would be 75, which means that the variance term is zero, so $EU^A = 0.5(75) + 0.5(75) = 75 >> -550$. And for $B, EU^B = 0.5(50) + 0.5(50) = 50 >> -575$. Clearly, both countries are better off. The trade reduces the volatility of consumption, which agents dislike (the point of the term, $-\frac{1}{2}\sum_i (c_i - \overline{c})^2$).

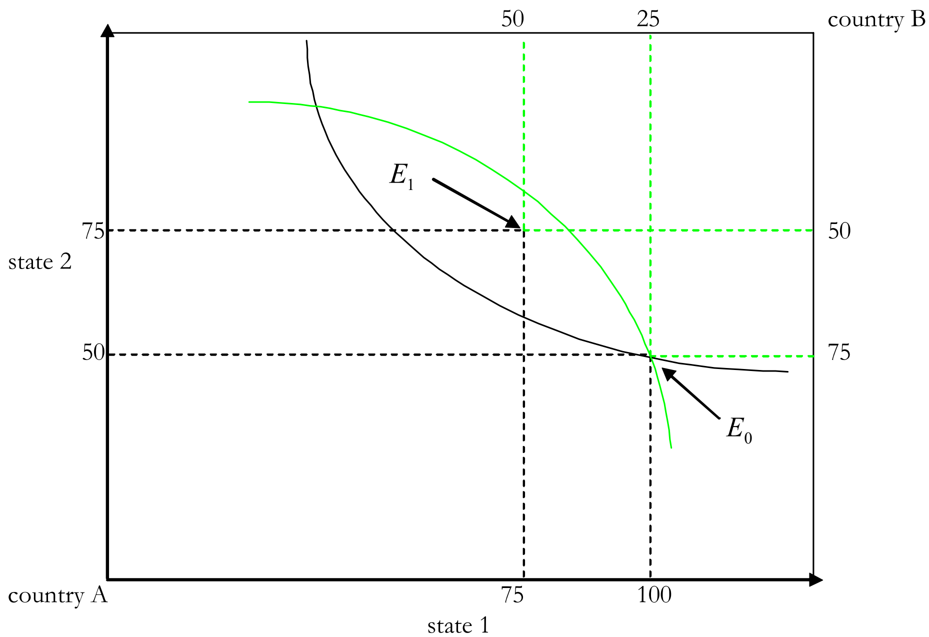



Figure 2: State Contingent Payoffs

(c) *Suppose that for country A the income levels in each state are now* $y_1^A = 125$, *and* $y_2^A = 25$. *What happens to expected utility under autarky? Suppose that for country B the income levels in each state are now* $y_1^B = 40$ *and* $y_2^B = 60$. *What happens to expected utility in country B under autarky? What does this tell us about the preferences of people in country A and B.*

**brief answer** In this case volatility has increased in country $A$ with no change in the mean value of consumption (mean preserving spread). Since there is increased volatility one expects that expected utility is lower than in part (a). For example,

$$EU^A = 0.5(125) + 0.5(25) - 0.5\left[(125-75)^2 + (25-75)^2\right] = -2425.$$

For country $B$, on the other hand, volatility has fallen, and so expected utility is now higher:

$$EU^B = 0.5(40) + 0.5(60) - 0.5\left[(40-50)^2 + (60-50)^2\right] = -50$$

This indicates that the preferences of people in both countries display risk aversion. Greater volatility of consumption lowers expected utility *even when there is no change in expected consumption.* In figure 3 I show the new endowment points. You can see that for country $A$ expected utility under autarky is clearly lower than before, while for country $B$ expected utility under autarky is higher.

(d) *Suppose expected utility is given by*

$$EU = \pi_1 c_1 + \pi_2 c_2 - \frac{\alpha}{2}\sum_i (c_i - \overline{c})^2 .$$

*If* $\alpha = 1$ *the problem is unchanged. What happens as* $\alpha$ *gets close to zero? What happens to the willingness to trade across states? Explain*

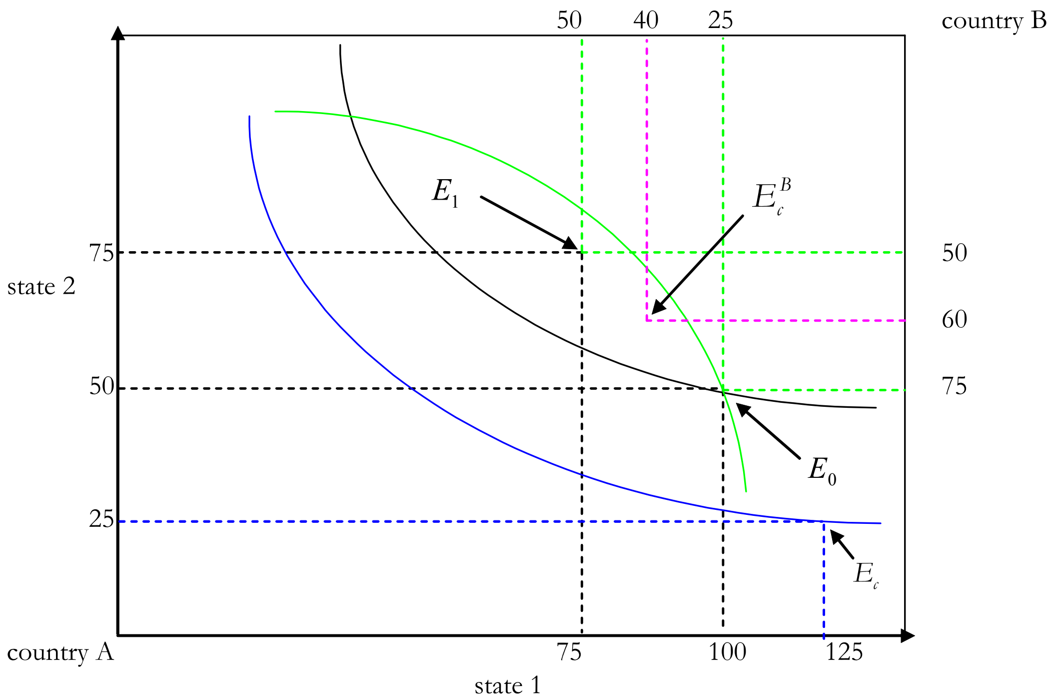



Figure 3: New Endowments

**brief answer** If $\alpha = 0$ then the last term drops off. This means that agents are indifferent to volatility and only care about expected consumption. In other words, they would be risk neutral. Bigger values of $\alpha$ means that agents are risk averse, they are willing to pay to reduce risks. Conversely, if $\alpha < 0$ then agents like volatile consumption – risk lovers. Back to the question, if $\alpha \to 0$ the willingness to trade decreases. There is no point to reducing risk.