Econ 434 Professor Ickes

Fall 2008

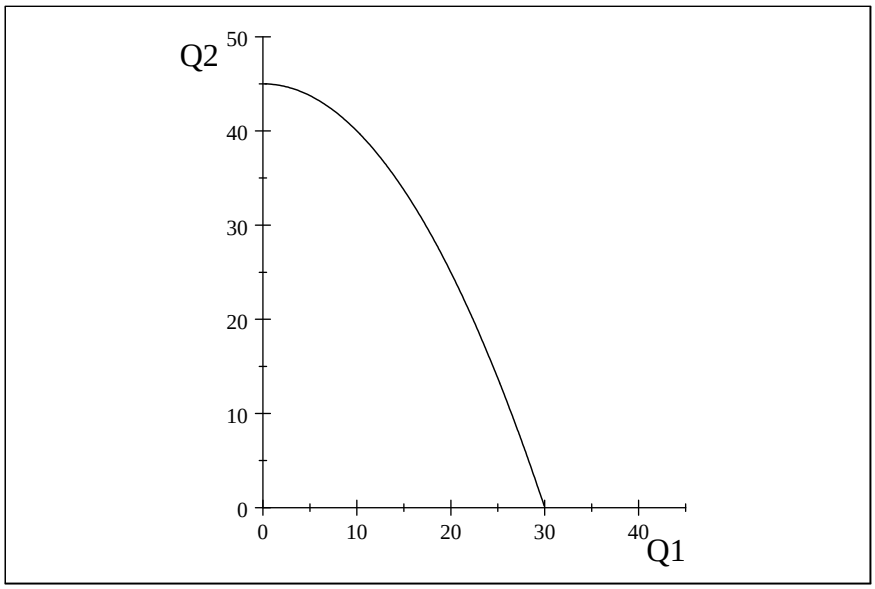
Homework Assignment #1: Answer Sheet

1. *Consider a small open economy that is endowed with* 30 *units of the consumption good in period one and* 0 *in period two. This is a one-good economy, but we also have investment. Specifically, period one output can be transformed into period two output according to the production frontier,*

$$Q_2 = 45 - .05Q_1^2. \tag{1}$$

Draw a two-period diagram with this endowment point and the production frontier suggested by (1) (hint: if none of the endowment is consumed in period one (fat chance of that!) what is Q_2*?). Does this production frontier exhibit diminishing returns to investment? Explain.*

brief answer See the figure below. Yes, it has diminishing returns. Slope of the production frontier is $-.1Q_1$, which clearly gets smaller as Q_1 gets smaller. You can also see that from an excel spreadsheet (at the end).



Production Possibilities

(a) *Suppose that the world interest rate is* 10%*. How would you find the optimal production point? Explain.*

brief answer Where the present value of production is maximized. That is, where $Q_1 + \frac{Q_2}{1+.1}$ is maximized. The excel spreadsheet shows that this is where $Q_1 = 11$. That is where the line with slope $-(1+r)$ is equal to the slope of the production frontier. Anywhere else has lower present value. This is evident in figure 1

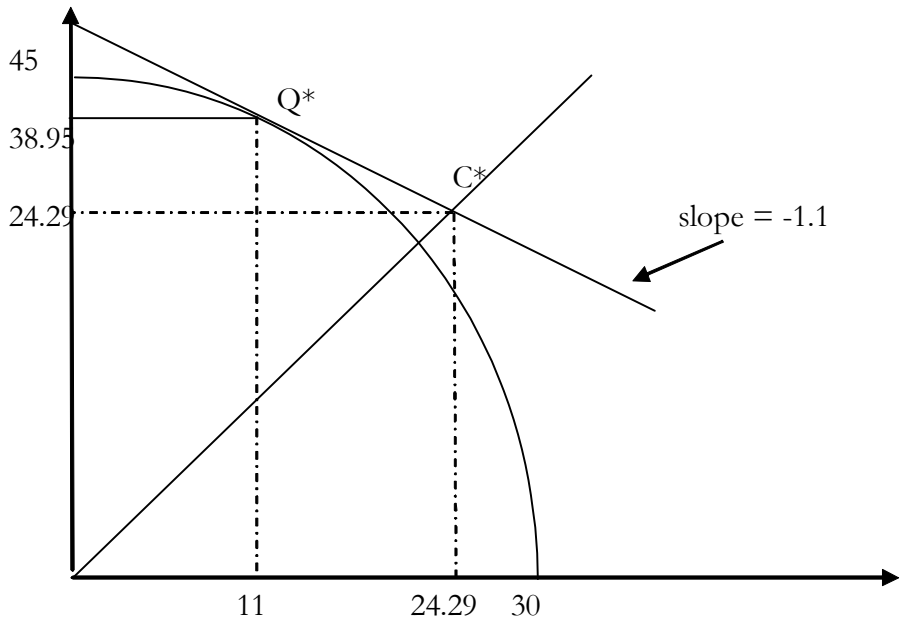


Figure 1: Optimal Production and Consumption Decision

(b) *What is the optimal production point [hint: how does Q_2 change when there is a change in Q_1]? If the interest rate fell to zero what would happen to the production point (in which direction would it move)?*

brief answer See the spreadsheet. It is clear that total value is highest when production $= 11$. If you knew calculus you could also proceed as follows, but that was not needed. Set $-(1+r) = -1.1$ to slope of the production frontier, $-.1Q_1$:

$$\begin{aligned} -1.1 &= -.1Q_1 \\ Q_1 &= 11 \\ Q_2 &= 45 - .05(11)^2 = 38.95 \end{aligned}$$

If the interest rate fell to zero then the value of future production rises, so we should produce more of it. From the spreadsheet we see that $Q_1 = 10, Q_2 = 40$. Or you could note that $-(1+r) = -(1+0) = -.1 = -.1Q_1 \Rightarrow Q_1 = 10, Q_2 = 40$. Production moves to the northwest along the frontier. This makes sense. A lower interest rate means that future production is even more profitable. So you produce more second period goods.

(c) *Suppose that for this country preferences are such that $\beta = \frac{1}{1+.1}$. Calculate the optimal consumption bundle [hint: recall the intertemporal budget constraint]. Will this country run a current account surplus or deficit in period one? Can you calculate its size? What is the value of the current account balance in period two?*

brief answer If $\beta = \frac{1}{1+r}$ then optimal consumption is equal in both periods, $C_1 = C_2$. The intertemporal budget constraint tells us that

$$C_1 + \frac{C_2}{1+r} = Q_1 + \frac{Q_2}{1+r}$$

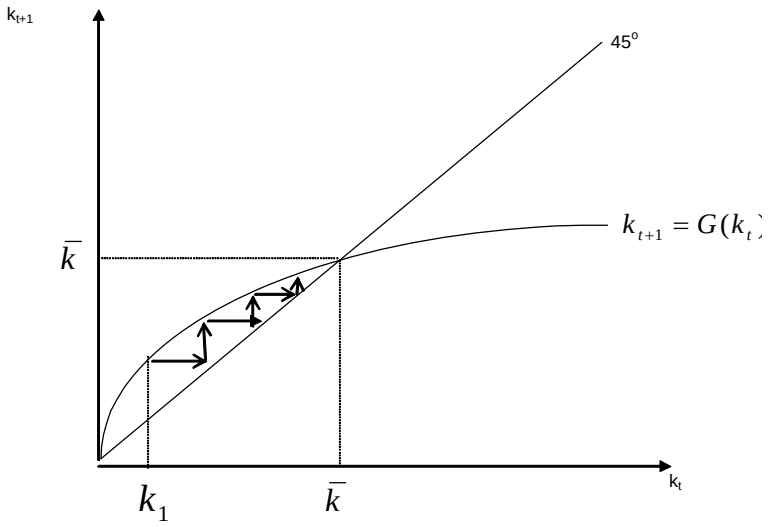


Figure 2: Recovery from disaster

but $C_1 + \frac{C_2}{1+r} = C_1 + \frac{C_1}{1+r} = C_1\left(1 + \frac{1}{1+r}\right)$, so

$$C_1\left(1 + \frac{1}{1+r}\right) = C_1\left(1 + \frac{1}{1+.1}\right) \approx C_1(1.91)$$

$$\begin{aligned} 1.91C_1 &= 11 + \frac{38.95}{1.1} \\ 1.91C_1 &= 46.40909 \\ C_1 &= C_2 = \frac{46.40909}{1.91} = 24.29 \end{aligned}$$

Since $C_1 > Q_1$ the country has a current account deficit in period one, $= 24.29 - 11 = 13.29$ In period two the current account surplus is $38.95 - 24.29 = 14.66$. Notice that the present value of the current account in period two $\frac{14.66}{1+.1} = 13.3$, so the present value of the current account balances over the two periods $= 0$, as we expect.

2. *Use the dynamic model to examine the impact of recovery from a natural disaster, like a big flood. Suppose that an industrialized country loses half of its capital stock in a flood (but, obviously, not its know-how). Examine what happens to the capital labor ratio, initially and over time, if the economy is closed. Compare this to what happens if the economy were open. In which case is the recovery faster? Why? Explain.*

brief answer The steady state capital-labor ratio is $\overline{k}$. The disaster destroys capital so we are at k_1. As the economic potential is unaffected the transition curve is unmoved. Capital accumulation causes the economy to recover as in figure 2. If the economy were open then capital could flow in from abroad in the wake of the disaster. The recovery would be quicker. At k_1 the marginal product of capital is higher than in the rest of the world, attracting capital. If the economy is small, then sufficient capital could flow in right

away. If not, then the adjustment would still take some time, but less than in the case of autarky.

3. *The current account deficit of the US today is roughly \$700 billion. The US share of world output is roughly 25% at market exchange rates. Total world savings is roughly \$12.5 trillion, which is about 23% of world GDP. Treat the rest of the world as one country. The rest of the world produces about 75% of world output. Total US savings is about \$1.8 trillion. Now suppose there was no home bias in investment (hint: what would this imply about the distribution of investment?). How much would the rest of the world invest in the US? How much would the US invest in the rest of the world? Are these investment levels higher or lower than what we actually observe? Would the current account deficit of the US be larger or smaller if there was no home bias (hint: remember that the current account balance is equal to the change in net foreign assets)?*

   brief answer ROW savings is total world savings minus US savings = \$12.5 − \$1.8 = \$10.7trillion. If there is no home bias then the ROW would invest in the US equal to the US share of world income = .25(10.7) = \$2. 675 trillion. We would invest .75(1.8) = \$1. 35 trillion in the ROW. These numbers are higher because we have home bias, which means we invest less than the optimal shares. The current account deficit of the US would be larger with no home bias. It would equal the change in net foreign assets which is equal to how much ROW buys of US assets minus what we buy of ROW assets. That is, $.25(S_{ROW}) - .75(S_{US}) = .25(10.7) - .75(1.8) = \$1.325$ which is larger than the current value of \$700 billion. Hence, if there was no home bias the rest of the world would be purchasing more US assets than they do now. If globalization means reduction of home bias this means we can expect the current account balance could worsen further in the near future (all else equal) from this source.

4. *Suppose that domestic investment and savings are given by:*

$$\begin{aligned} I_{US} &= 100 - 3r_{us} \\ S_{US} &= 20 + 6r_{us} \end{aligned}$$

   and that investment and savings in the rest of the world are given by:

$$\begin{aligned} I_{ROW} &= 120 - 7r_{\text{row}} \\ S_{ROW} &= 40 + 6r_{\text{row}} \end{aligned}$$

   (a) *Suppose the US is a closed economy. What will the interest rate be in the US? What will the interest rate be in the rest of the world?*

   brief answer In a closed economy $I_{US} = S_{US}$, so $20 + 6r_{us} = 100 - 3r_{us}$. Thus,

$$\begin{aligned} 9r_{US} &= 80 \\ r_{US} &= \frac{80}{9} = 8.889 \end{aligned}$$

   In the ROW we have

$$\begin{aligned} 120 - 7r_{\text{row}} &= 40 + 6r_{\text{row}} \\ 80 &= 13r_{\text{row}} \Longrightarrow r_{\text{row}} = \frac{80}{13} = 6.154 \end{aligned}$$

(b) *Suppose that the US opens up to the rest of the world. What will the world interest rate be equal to?*

brief answer Market-clearing now requires that the world interest rate satisfy:

$$I_{US} + I_{\text{row}} = S_{US} + S_{\text{row}}$$

or

$$\begin{aligned} 100 - 3r^* + 120 - 7r^* &= 20 + 6r^* + 40 + 6r^* \\ 100 + 120 - 20 - 40 &= r^*(6 + 6 + 3 + 7) \\ 160 &= 22r^* \\ r^* &= 160/22 = 7.2727 \end{aligned}$$

We can note that r^* is less than the US autarky rate but above the ROW autarky rate ($6.154 < 7.2727 < 8.889$) , which we know must be the case.

(c) *At the equilibrium world interest rate calculate net savings in the US and the ROW. Will the US have positive net savings?*

brief answer We know that at r^* the US will have negative net savings. This follows because at $r_{US} = 8.889$ we had $I_{US} = S_{US}$, and $r^* < 8.889$, so savings will decrease and investment will be higher. So we know the answer. But we are supposed to calculate it. We can substitute into the savings and investment functions to obtain the answer. For the US, we have:

$$\begin{aligned} I_{US} &= 100 - 3(7.2727) = 78.182 \\ S_{US} &= 20 + 6(7.2727) = 63.636 \end{aligned}$$

so the $CA_{US} = 63.636 - 78.182 = -14.546 < 0$. For ROW we have:

$$\begin{aligned} I_{ROW} &= 120 - 7(7.2727) = 69.0911 \\ S_{ROW} &= 40 + 6(7.2727) = 83.636 \end{aligned}$$

so $CA_{ROW} = 83.636 - 69.0911 = 14.545$. Fortunately for us, these sum to zero (subject to rounding error). That tells us we have the right answer.

(d) *Suppose that US savings increases. Specifically, suppose it shifts upwards by* 20 *at any* r*. What happens to the autarky interest rate in the US? What happens to the equilibrium world interest rate if the US is open?*

brief answer The autarky rate must fall as this creates an excess of savings over investment in the US at the old autarky rate. We see that now we have $40 + 6r_{us} = 100 - 3r_{us} \Longrightarrow 9r_{us} = 60 \Longrightarrow r_{us} = \frac{60}{9} = 6.6667$, which is, indeed, less than 8.889. If capital markets were liberalized then the world interest rate will be lower. As the US autarky rate is now 6.667, we know that r^* must be between 6.154 and 6.667.

(e) *Calculate net savings in the US and the ROW at the new equilibrium world interest rate. Will the US have positive net savings?*

brief answer Follow the same procedure as in parts (b) and (c) but with the new US savings function:

$$\begin{aligned} 100 - 3r^* + 120 - 7r^* &= 40 + 6r^* + 40 + 6r^* \\ 100 + 120 - 40 - 40 &= r^*(6 + 6 + 3 + 7) \\ 140 &= 22r^* \\ r^* &= 140/22 = 6.3636 \end{aligned}$$

which does satisfy our last assertion in part (d). Now just use this value as we did in part (c):

$$\begin{aligned} I_{US} &= 100 - 3(6.3636) = 80.909 \\ S_{US} &= 40 + 6(6.3636) = 78.182 \end{aligned}$$

so $CA_{US} = 78.182 - 80.909 = -2.727$, so the US has a very small current account deficit. You can guess what the answer is for ROW. But why not just calculate to make sure.

$$\begin{aligned} I_{ROW} &= 120 - 7(6.3636) = 75.455 \\ S_{ROW} &= 40 + 6(6.3636) = 78.182 \end{aligned}$$

so $CA_{\text{row}} = 2.727$, which is satisfying, since $-2.727 + 2.727 = 0$.

Q1	Q2	change in Q2/ change in Q1	-(1+r)	present value of production	present value of production with r = 0
1	44.95		-1.1	41.86	45.95
2	44.8	-0.15	-1.1	42.73	46.8
3	44.55	-0.25	-1.1	43.50	47.55
4	44.2	-0.35	-1.1	44.18	48.2
5	43.75	-0.45	-1.1	44.77	48.75
6	43.2	-0.55	-1.1	45.27	49.2
7	42.55	-0.65	-1.1	45.68	49.55
8	41.8	-0.75	-1.1	46.00	49.8
9	40.95	-0.85	-1.1	46.23	49.95
10	40	-0.95	-1.1	46.36	50 ← maximum with r = 0
11	38.95	-1.05	-1.1	46.41 ←	49.95 maximum with r = 10%
12	37.8	-1.15	-1.1	46.36	49.8
13	36.55	-1.25	-1.1	46.23	49.55
14	35.2	-1.35	-1.1	46.00	49.2
15	33.75	-1.45	-1.1	45.68	48.75
16	32.2	-1.55	-1.1	45.27	48.2
17	30.55	-1.65	-1.1	44.77	47.55
18	28.8	-1.75	-1.1	44.18	46.8
19	26.95	-1.85	-1.1	43.50	45.95
20	25	-1.95	-1.1	42.73	45
21	22.95	-2.05	-1.1	41.86	43.95
22	20.8	-2.15	-1.1	40.91	42.8
23	18.55	-2.25	-1.1	39.86	41.55
24	16.2	-2.35	-1.1	38.73	40.2
25	13.75	-2.45	-1.1	37.50	38.75
26	11.2	-2.55	-1.1	36.18	37.2
27	8.55	-2.65	-1.1	34.77	35.55
28	5.8	-2.75	-1.1	33.27	33.8
29	2.95	-2.85	-1.1	31.68	31.95
30	0	-2.95	-1.1	30.00	30

Figure 3: Spreadsheet for Problem 1