Econ 434 Professor Ickes

Fall 2009

Homework Assignment #1: Answer Sheet

This assignment is due on Tuesday, September 15, at the beginning of class (or sooner).

1. *Consider the following returns data for two economies, Macrodonia and Microdonia:*

	Panel A		Panel B				
Year	*Macrodonia* %	*Microdonia* %	x %	y %	*Return* %	*Risk* %
1994	−9.5	28.1	100	0	13.38	23.15
1995	56.7	69.4	90	10	13.85	21.53
1996	6.2	21.1	80	20	14.31	20.52
1997	36.6	25.3	70	30	14.77	20.2
1998	14.3	10.7	60	40	15.24	20.61
1999	−12.5	67.7	50	50	15.7	21.71
2000	42.6	−28.6	40	60	16.17	23.4
2001	−0.5	−34.0	30	70	16.64	25.56
2002	−12.1	−13.9	20	80	17.1	28.09
2003	12.0	34.5	10	90	17.57	30.89
average	13.38	18.03	0	100	18.03	33.90
standard deviation	23.15	33.90				
correlation 0.127						

(a) *Calculate the average return and the standard deviation of returns for each country over this period (use a calculator or an excel spreadsheet, which has a built-in formula for this, or the excel program I provided for the class in my lecture). Also calculate the correlation of returns across the countries. The latter is denoted by ρ_{ij}.*

brief answer see table.

(b) *The expected return for a portfolio with two assets is given by $R_p = xR_i + yR_j$. Let i be Macrodonia and j be Microdonia, and let x be the share of the portfolio invested in Macrodonia assets, and y the share in Microdonia assets. Calculate the expected returns for each of the ten portfolios given by the weights in panel B of the table.*

brief answer see table.

(c) *The risk of a portfolio is given by its standard deviation, σ. For a portfolio of two assets this is given by the formula:*

$$\sigma_P = \left(x^2\sigma_i^2 + y^2\sigma_j^2 + 2xy\sigma_i\sigma_j\rho_{ij}\right)^{\frac{1}{2}}$$

Use this formula to calculate the riskiness of each of the ten portfolios in panel B of the table(remember the excel program).

brief answer see table.

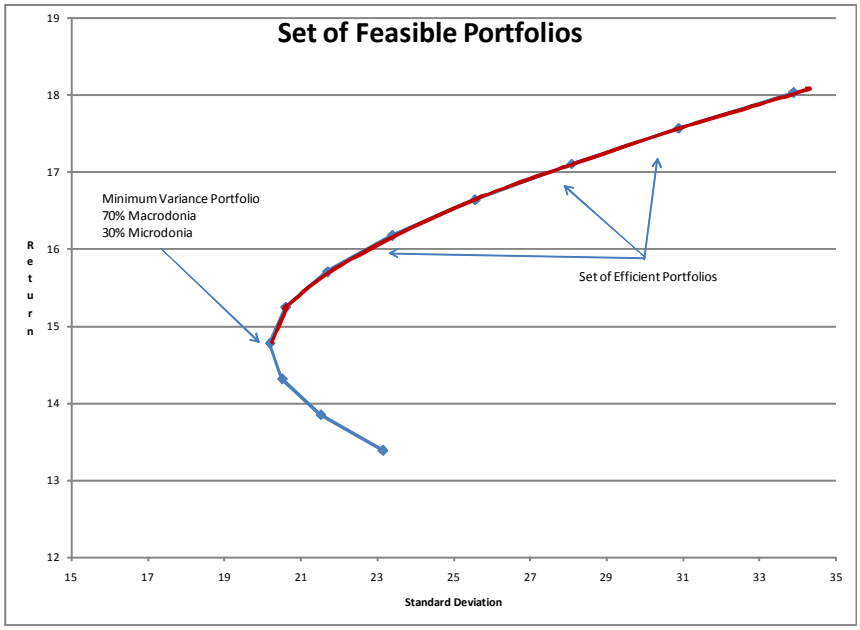

Figure 1:

(d) *Using your calculations of the risk and return of the ten portfolios make a graph of the feasible set of portfolios. Which portfolio has the minimum risk?*

(e) *Label the efficient set of portfolios in your graph. Explain why a portfolio that is comprised only of home assets would be a poor choice for residents of either country.*

brief answer A portfolio comprised of %100 Macrodonia assets has $R_p = 13.38$ and $\sigma_p = 23.15$. This is not efficient – you could get a higher return with the same amount of risk. For example, 40% Microdonia 60% Microdonia yields $R_p = 16.17$ and has the same level of risk. A portfolio comprised of all Microdonia assets would be optimal only for investors who did not care about risk, only return. If an investor was risk neutral her indifference curves would be horizontal (can you explain why) and thus would hold only Microdonia assets. An investor that was risk averse would choose a portfolio somewhere on the efficient frontier, but most likely a diversified portfolio.

2. *Consider a small open economy that is endowed with* 20 *units of the consumption good in period one and* 0 *in period two. This is a one-good economy, but we also have investment. Specifically, period one output can be transformed into period two output according to the production frontier,*

$$Q_2 = 100 - .25Q_1^2. \tag{1}$$

Draw a two-period diagram with this endowment point and the production frontier suggested by (1) (hint: if none of the endowment is consumed in period one (fat chance of that!) what is Q_2?). Does this production frontier exhibit diminishing returns to investment? Explain.

brief answer *See figure. You can see diminishing returns as the production frontier gets flatter as more investment takes place in the first period.*

(a) Suppose that the world interest rate is 9%. How would you find the optimal production point (a graphical explanation is sufficient)? See figure 2

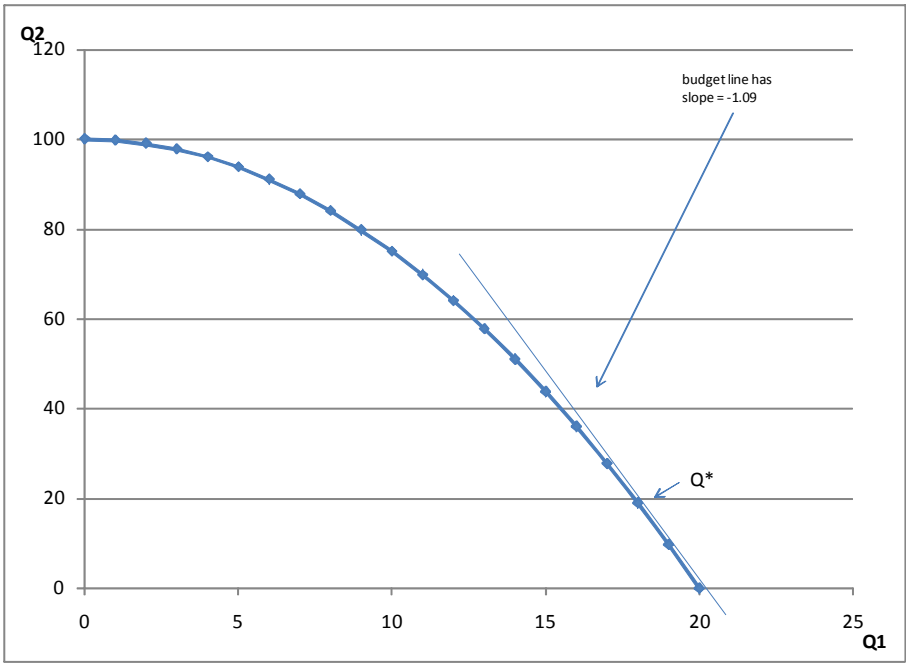

Figure 2: Production Possibility Frontier

brief answer (b) *What is the optimal production point [hint: you can use an excel spreadsheet to calculate Q_2 as a function of Q_1]? If the interest rate fell to zero what would happen to the production point?*

brief answer We can calculate the rate of return to investment as $Q_2 - Q_1$ and compare this to the interest rate of ..09. We getWe can see that if we invest two units in period one the return exceeds 9%, but if we invest 3 units the marginal return is only 8.75%. So the optimal production point is at $Q_1 = 18, Q_2 = 19$. You can also see this graphically by drawing a line with the slope of -1.09 and seeing that it is tangent at that point. Or, if you knew calculus, you could calculate that $\frac{-dQ_2}{dQ_1} = .5Q_1$ If we set $.5Q_1 = .09$ then obviously $Q_1 = \frac{.09}{.5} = 18$.

(c) *Suppose that for this country preferences are such that $\beta = \frac{1}{1+.09}$. Calculate the optimal consumption bundle [hint: recall the intertemporal budget constraint]. Will this country run a current account surplus or deficit in period one? Can you calculate its size? What is the value of the current account balance in period two?*

brief answer If $\beta = \frac{1}{1+r}$ then optimal consumption is equal in both periods, $C_1 = C_2$. The intertemporal budget constraint tells us that

$$C_1 + \frac{C_2}{1+r} = Q_1 + \frac{Q_2}{1+r}$$

but $C_1 + \frac{C_2}{1+r} = C_1 + \frac{C_1}{1+r} = C_1\left(1 + \frac{1}{1+r}\right)$, so

$$C_1\left(1 + \frac{1}{1+r}\right) = C_1\left(1 + \frac{1}{1+.09}\right) \approx C_1(1.92)$$

Q1	Q2	rate of return (pct)
20	0	
19	9.75	0.0975
18	19	0.0925
17	27.75	0.0875
16	36	0.0825
15	43.75	0.0775
14	51	0.0725
13	57.75	0.0675
12	64	0.0625
11	69.75	0.0575
10	75	0.0525
9	79.75	0.0475
8	84	0.0425
7	87.75	0.0375
6	91	0.0325
5	93.75	0.0275
4	96	0.0225
3	97.75	0.0175
2	99	0.0125
1	99.75	0.0075
0	100	0.0025

Figure 3:

$$\begin{aligned} 1.92C_1 &= 18 + \frac{19}{1.09} \\ 1.92C_1 &= 35.43 \\ C_1 &= C_2 = \frac{35.43}{1.92} = 18.453 \end{aligned}$$

Since $C_1 > Q_1$ the country has a current account deficit in period one, $= 18.453 - 18 = .453$ In period two the current account surplus is $19 - 18.453 = 0.547$ .Notice that the present value of the current account in period two $\frac{.547}{1+.09} = 0.501\,83 = 0.475\,23$ so the present value of the current account balances over the two periods $= 0$, as we expect (given rounding error).

3. *Suppose that domestic investment and savings are given by:*

$$\begin{aligned} I_{US} &= 200 - 4r_{us} \\ S_{US} &= 35 + 7r_{us} \end{aligned}$$

and that investment and savings in the rest of the world are given by:

$$\begin{aligned} I_{ROW} &= 120 - 7r_{\text{row}} \\ S_{ROW} &= 40 + 6r_{\text{row}} \end{aligned}$$

(a) *Suppose the US is a closed economy. What will the interest rate be in the US? What will the interest rate be in the rest of the world?*

brief answer In a closed economy $I_{US} = S_{US}$, so $200 - 4r_{us} = 35 + 7r_{us}$. Thus,

$$\begin{aligned} 11r_{US} &= 165 \\ r_{US} &= \frac{165}{11} = 15 \end{aligned}$$

In the ROW we have

$$\begin{aligned} 120 - 7r_{\text{row}} &= 40 + 6r_{\text{row}} \\ 80 &= 13r_{\text{row}} \Longrightarrow r_{\text{row}} = \frac{80}{13} = 6.154 \end{aligned}$$

(b) *Suppose that the US opens up to the rest of the world. What will the world interest rate be equal to?*

brief answer

$$I_{US} + I_{\text{row}} = S_{US} + S_{\text{row}}$$

or

$$\begin{aligned} 200 - 4r^* + 120 - 7r^* &= 35 + 7r^* + 40 + 6r^* \\ 200 + 120 - 35 - 40 &= r^*(4 + 7 + 6 + 7) \\ 245 &= 24r^* \\ r^* &= 245/24 = 10.21 \end{aligned}$$

(c) *At the equilibrium world interest rate calculate net savings in the US and the ROW. Will the US have positive net savings?*

brief answer We know that at r^* the US will have negative net savings. This follows because at $r_{US} = 15$ we had $I_{US} = S_{US}$, and $r^* < 15$, so savings will decrease and investment will be higher. So we know the answer. But we are supposed to calculate it. We can substitute into the savings and investment functions to obtain the answer. For the US, we have:

$$\begin{aligned} I_{US} &= 200 - 4(10.21) = 159.2 \\ S_{US} &= 35 + 7(10.21) = 106.47 \end{aligned}$$

so the $CA_{US} = 106.47 - 159.2 = -52.73 < 0$. For ROW we have:

$$\begin{aligned} I_{ROW} &= 120 - 7(10.21) = 48.53 \\ S_{ROW} &= 40 + 6(10.21) = 101.26 \end{aligned}$$

so $CA_{ROW} = 101.26 - 48.53 = 52.73$. Fortunately for us, these sum to zero (subject to rounding error). That tells us we have the right answer.

(d) *Suppose that US savings increases. Specifically, suppose it shifts upwards by* 40 *at any* r*. What happens to the autarky interest rate in the US? What happens to the equilibrium world interest rate if the US is open?*

brief answer The autarky rate must fall as this creates an excess of savings over investment in the US at the old autarky rate. We see that now we have $75 + 7r_{us} = 200 - 4r_{us} \Longrightarrow 11r_{us} = 125 \Longrightarrow r_{us} = 11.364$, which is, indeed, less than 15. If capital markets were liberalized then the world interest rate will be lower. As the US autarky rate is now 6.667, we know that r^* must be between 11.36 and 6.154. You can calculate it as in problem *b*.

(e) *Calculate net savings in the US and the ROW at the new equilibrium world interest rate. Will the US have positive net savings?*

brief answer Follow the same procedure as in parts (b) and (c) but with the new US savings function:

$$\begin{aligned} 200 - 4r^* + 120 - 7r^* &= 40 + 6r^* + 75 + 7r^* \\ 200 + 120 - 40 - 75 &= r^*(4 + 7 + 6 + 7) \\ 205 &= 24r^* \\ r^* &= 205/24 = 8.54 \end{aligned}$$

which does satisfy our last assertion in part (d). Now just use this value as we did in part (c):

$$\begin{aligned} I_{US} &= 200 - 4(8.54) = 165.84 \\ S_{US} &= 75 + 7(8.54) = 134.78 \end{aligned}$$

so $CA_{US} = 134.78 - 165.84 = -31.06$, so the US has a very small current account deficit. You can guess what the answer is for ROW. But why not just calculate to

make sure.$40 + 6(8.54) = 91.\,24$

$$\begin{aligned} I_{ROW} &= 120 - 7(8.54) = 60.22 \\ S_{ROW} &= 40 + 6(8.54) = 91.24 \end{aligned}$$

so $CA_{\text{row}} = 91.24 - 60.22 = 31.\,02$, which is satisfying, since $-31.06 + 31.02 = 0$ (given rounding error).

4. *Consider a two-country world with one time period. There is one good, but income is stochastic. There are two states of nature, denoted* 1 *and* 2*. The probability of each state is given by* $\pi_1 = \pi_2 = 0.5$*. The identical agents in each country maximize expected utility, given by*

$$\begin{aligned} EU &= \pi_1 c_1 + \pi_2 c_2 - \frac{1}{2}\left[(c_1 - \overline{c})^2 + (c_2 - \overline{c})^2\right] \\ &= \pi_1 c_1 + \pi_2 c_2 - \frac{1}{2}\sum_i (c_i - \overline{c})^2 \end{aligned} \tag{2}$$

where c_i *(*$i = 1, 2$*) is the state that actually occurs, and* $\overline{c} = \frac{c_1 + c_2}{2}$ *is the average level of consumption. In state* 1 *income in country A (*y_1^A*) is* 200 *and in state* 2, $y_2^A = 100$*. In country B,* $y_1^B = 30$ *and* $y_2^B = 70$*.*

(a) *Provide a verbal explanation of these preferences (2).*

brief answer Expected utility depends on average consumption across states minus a penalty for variance in consumption. The greater the variance the lower is expected utility, so this means the agent is risk averse.

(b) *Suppose that there is no trade across countries. Calculate expected utility in each country.*

brief answer If autarky then we have $c_i = y_i$ in each state for each country.

$$\begin{aligned} EU^A &= 0.5(200) + 0.5(100) - 0.5\left[(200 - 150)^2 + (100 - 150)^2\right] = -2350 \\ EU^B &= 0.5(30) + 0.5(70) - 0.5\left[(30 - 50)^2 + (70 - 50)^2\right] = -350 \end{aligned}$$

(c) *Suppose the two countries can trade an asset that has a state-contingent payoff. The price of the asset is unity. Can these two economies trade across states to increase their expected utility? Explain. What would the contract look like (who trades how much in each state)?*

brief answer Obviously country A would like to offer some of the good in state 1 to compensate for the risk of state 2. For country B it is just the opposite. Suppose that country A trades 20 to country B in state 1 in exchange for the receipt of 20 in state 2 (I picked this for ease of computation, there are other beneficial trades like this). This is mutually advantageous, and given that each state is equally likely has great benefits. Consider that for country B its consumption now is 50 regardless of the state. So $EU^B = 50 > -350$!!! For country A not all risk is eliminated, but some is, so $EU^A = 0.5(180) + 0.5(120) - 0.5\left[(180 - 150)^2 + (120 - 150)^2\right] = -750 > -2350$, so$EU^A$ also rises. See the figure, where point E_1 represents this new consumption point. Notice that people in both countries are on higher indifference curves.

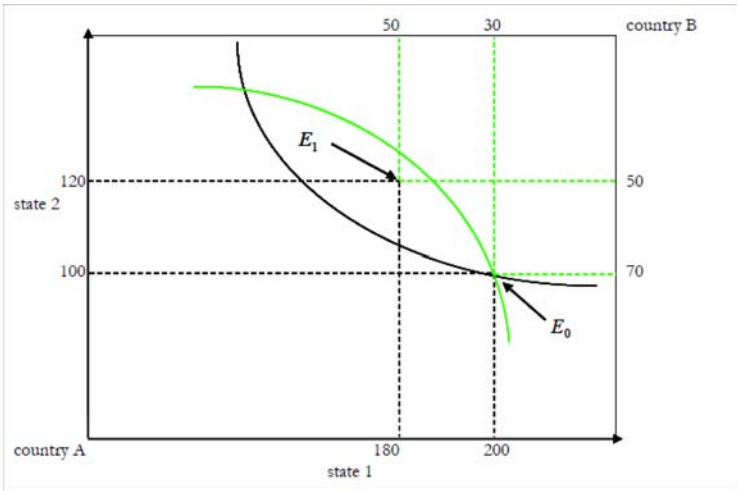

Figure 4:

(d) *Suppose that for country A the income levels in each state are now* $y_1^A = 225$, *and* $y_2^A = 75$. *What happens to expected utility under autarky? Suppose that for country B the income levels in each state are now* $y_1^B = 45$ *and* $y_2^B = 55$. *What happens to expected utility in country B under autarky? What does this tell us about the preferences of people in country A and B..*

brief answer In this case volatility has increased in country A with no change in the mean value of consumption (mean preserving spread). Since there is increased volatility one expects that expected utility is lower than in part (a). Thus:

$$EU^A = 0.5(225) + 0.5(75) - 0.5\left[(225-150)^2 + (75-150)^2\right] = -5475.$$

So expected utility has fallen. For country B, on the other hand, volatility has fallen, and so expected utility is now higher:

$$EU^B = 0.5(45) + 0.5(55) - 0.5\left[(45-50)^2 + (55-50)^2\right] = 25$$

This indicates that the preferences of people in both countries display risk aversion. Greater volatility of consumption lowers expected utility *even when there is no change in expected consumption.*

(e) *Suppose expected utility is given by*

$$EU = \pi_1 c_1 + \pi_2 c_2 - \frac{\alpha}{2}\sum_i (c_i - \overline{c})^2 .$$

If $\alpha = 1$ *the problem is unchanged. What happens as* α *gets close to zero? What happens to the willingness to trade across states? What if* $\alpha > 1$*? Explain*

brief answer If $\alpha = 0$ then the last term drops off. This means that agents are indifferent to volatility and only care about expected consumption. In other words, they would be risk neutral. Bigger values of α means that agents are risk averse, they are willing to pay to reduce risks. Conversely, if $\alpha < 0$ then agents like volatile

consumption – risk lovers. Back to the question, if $\alpha \to 0$ the willingness to trade decreases. There is no point to reducing risk. If $\alpha > 1$ the agent is risk loving – expected utility rises if consumption is more variable (think of gamblers). In this case there may be trade that is profitable if it *increases* consumption volatility.