Econ 434 Professor Ickes

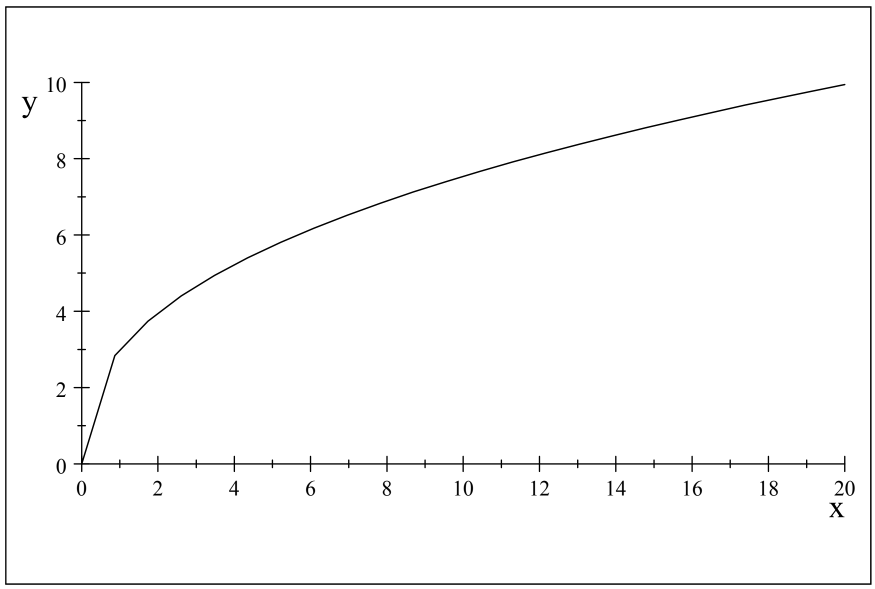
Fall 2006

Homework Assignment #2: Answer Sheet

1. *Consider the dynamic model. There are two countries, US and Japan. They have identical production functions, $y = Ak^{\beta}$, where $A = 100$ in both countries, and capital's share, $\beta = 0.4$ in each country. Suppose that the savings rate $(1-\alpha)$ is .05 in the US and .20 in Japan.*

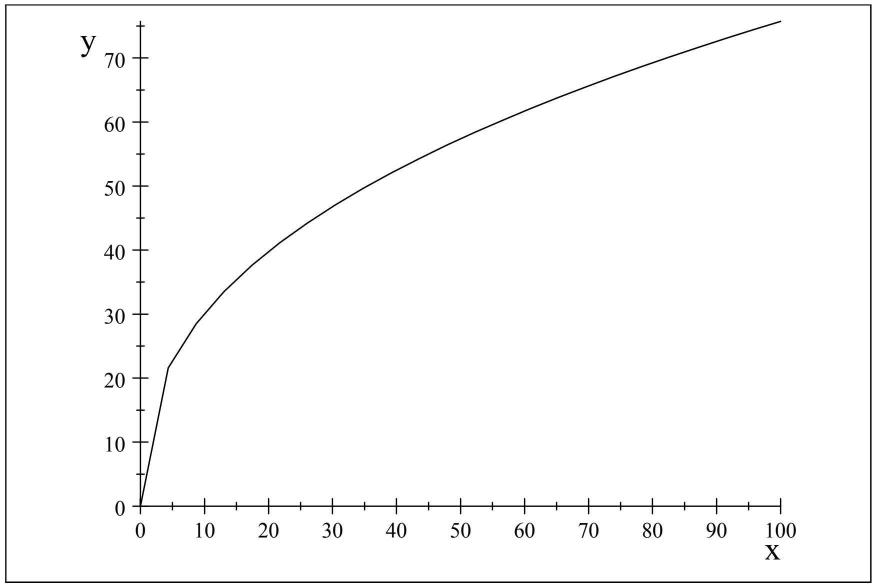
   (a) *Draw the transition curve for each country.*

   brief answer The transition equation is $k_{t+1} = (1-\alpha)(1-\beta)A_t k_t^{\beta}$. So for the US we have $k_{t+1} = (1-.95)(1-.4)(100)k_t^{.4}$



US Transition Equation: $(1-.95)(1-.4)(100)k_t^{.4}$

and for Japan we have $k_{t+1} = (1-.8)(1-.4)(100)k_t^{.4}$



Transition Equation for Japan, $(1-.8)(1-.4)(100)k_t^{.4}$

(b) *Under autarky compute the steady state capital-labor ratios for the US and Japan. Which country will have higher per-capita output? Explain.*

brief answer We are working with the steady state of the model (see lecture note, page 25, equation 24). The key expression for the steady state capital-labor ratio is

$$\overline{k} = [(1-\alpha)(1-\beta)A_t]^{\frac{1}{1-\beta}}$$

We thus merely substitute and calculate. Thus for the US, we have

$$\overline{k}_{US} = [(1-.95)(1-.4)100]^{\frac{1}{1-.4}} = 6.2403$$

and for Japan,

$$\overline{k}_J = [(1-.8)(1-.4)100]^{\frac{1}{1-.4}} = 62.898$$

To get per-capita output we substitute $\overline{k}$ into the production function. So for the US, we have $\overline{y}_{US} = 100(6.2403)^{.4} = 208.01$, and for Japan we have $\overline{y}_J = 100(62.898)^{.4} = 524.15$.

(c) *Suppose that capital markets are integrated, and assume that the US has twice the population of Japan. What happens to k in each country (i.e., what happens when integration occurs). What will the steady-state capital labor ratio be under complete integration?*

brief answer Since the rate of return is higher in the US, savings will flow from Japan to the US. This means that k will rise in the US, and will fall in Japan. We have $N^{US} = 2N^J$, so $\overline{\alpha} = \frac{1}{3}(.8) + \frac{2}{3}(.95) = 0.9$. So the world steady-state capital-labor ratio is given by:

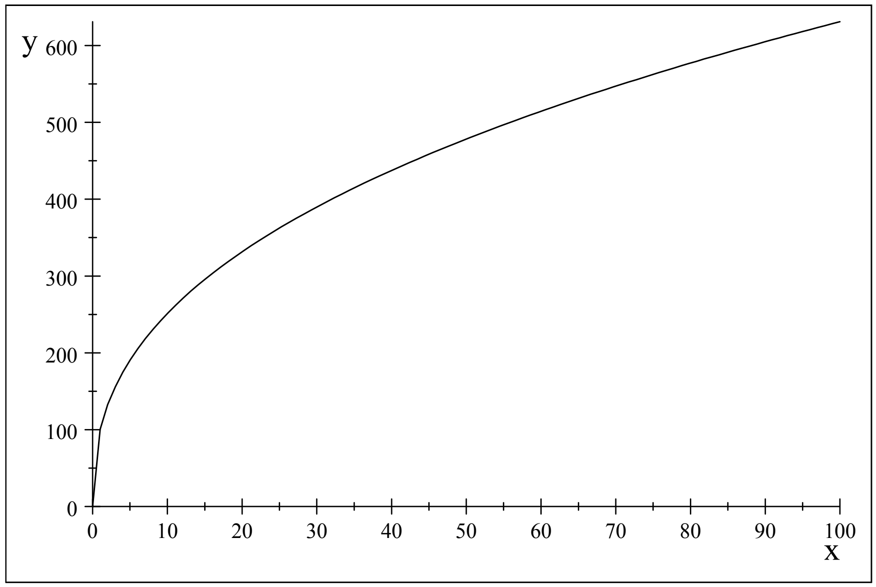
$$\overline{k}_W = [(1-.9)(1-.4)100]^{\frac{1}{1-.4}} = 19.812$$

1. *What will per-capita output be in this steady state? How does it compare to world output under autarky?*

brief answer $\overline{y}^W = 100(19.812)^{.4} = 330.20$. Under autarky, output per worker is 208.01 in the US and 524.15 in Japan. Since the US is twice as large as Japan, world output under autarky is $\frac{1}{3}(524.15) + \frac{2}{3}(208.01) = 313.39$. So world output rises with liberalization.

2. *What will the rate of return to capital be in the new steady state [hint: find an expression for the marginal product of capital]? How does this compare to the rate of return to capital under autarky in each economy?*

brief answer The marginal product of capital $= \beta A k^{\beta-1}$, so we have



Production Function

$$f_{\overline{k}_W} = .4(100)(19.812)^{(.4-1)} = 6.6666$$

. For the US and Japan we have, $f_k^{US} = .4(100)(6.240)^{(.4-1)} = 13.334$, and $f_k^J = .4(100)(62.898)^{(.4-1)} = 3.3333$. You can see how much steeper the production function is at $x = 6.24$ than at $x = 62.89$.

3. *In the integrated capital markets steady state, which economy will have positive net foreign assets and which will have negative? Explain.*

brief answer Japan will have positive NFA and the US will have negative, since Japan is investing in the US.

4. *What is the level of net foreign assets for Japan and for the US? How would you figure this out?*

brief answer Recall that the steady state capital labor ratio is 19.812. Then savings in Japan is given by $k_{t+1} = (1 - .8)(1 - .4)100(19.812)^{.4} = 39.623$, so $39.623 - 19.812 = 19.811$ must be invested abroad, Japan's level of NFA. We can see that this is correct by noting that for the US, we have $k_{t+1} = (1 - .95)(1 - .4)100(19.812)^{.4} = 9.9059$. So each American as -9.905 net foreign assets per worker, while each Japanese has 19.812 in nfa. But recall that there are two Americans for every Japanese, and $2(9.9059) = 19.812$.

(d) *Suppose that productivity in the US rises permanently to 150. What will happen to the steady state world capital-labor ratio? Explain.*

brief answer We know that this will raise the return to investing, so the capital-labor ratio will rise. Before, we had $r_{US} = .4(100)(19.812)^{.4-1} = .4(100)(19.812)^{.4-1} = r_J$. Now we have $r_{US} = .4(150)(19.812)^{.4-1} = 9.9999$ which is greater than $r_J = .4(100)(19.812)^{.4-1} = 6.6666$. So capital will flow from Japan to the US to equalize rates of return. The new equilibrium requires, $.4(100)k_J^{.4-1} = .4(150)k_{US}^{.4-1}$, or $\left(\frac{k_{us}}{k_J}\right)^{.4-1} = \frac{.4(100)}{.4(150)}$ or $\frac{k_{us}}{k_J} = \left(\frac{.4(100)}{.4(150)}\right)^{\frac{1}{.4-1}} = 1.9656$, that is the capital-labor ratio in the US will be 1.965 larger than in Japan. So about 1/3 of the world capital stock is in Japan, the rest in the US. Thus, the world capital labor ratio, $\overline{k}^W = (1-.9)(1-.4)[\left(\frac{1}{1+1.965}\right)100 + \left(1-\frac{1}{1+1.965}\right)150]k^{.4}$, so $\overline{k}_W = [(1-.9)(1-.4)[\left(\frac{1}{1+1.965}\right)100 + \left(1-\frac{1}{1+1.965}\right)150]]^{\frac{1}{1-.4}}$, and note that $\left(\frac{1}{1+1.965}\right)100 + \left(1-\frac{1}{1+1.965}\right)150 = 133.14$, so $\overline{k}_W = [(1-.9)(1-.4)133.14]^{\frac{1}{1-.4}} = 31.923$.

1. *In the new equilibrium what will be the relative sizes of the two economies (say, in terms of their capital-labor ratios)?*

   brief answer We already showed that the US capital-labor ratio is 1.965 times larger than Japan. We can further show that $k_w = 31.923 = (1/3k)_j + (2/3)k_{us} = (1/3)k_j + (2/3)(1.956)k_j$, so $k_j[1/3 + (2/3)1.965] = 31.923$, or $k_j = \frac{31.923}{1/3+(2/3)1.965} = 19.426$, so $k_{us} = 1.965(19.426) = 38.172$. One could then show that output in the US is $y_{us} = 150(38.172)^{.4} = 643.86$, and in Japan we have $k_j = 100(19.426)^{.4} = 327.61$. We can see that productivity differences lead to large output differences, even though Japan saves more than the US.