Econ 434 Professor Ickes

Fall 2008

Homework Assignment #2: Answer Sheet

1. *Suppose that the price level in the home country is given by $P = P_n^{\alpha} P_t^{1-\alpha}$, where P_t is the price of traded goods, and α is the share of non-traded goods in the domestic price index, and similarly $P^* = P_n^{*\alpha} P_t^{*1-\alpha}$ for the foreign country. Suppose that tradables have a common price of 1 in both countries. Show how the ratio of home to foreign prices depends on the relative price of non-traded goods (e.g., derive a simple expression for this).*

   brief answer This part is trivial and is only to set up the rest. $P = (1)^{1-\alpha} P_n^{\alpha} = P_n^{\alpha}$ and likewise $P^* = (P_n^*)^{\alpha}$ for the foreign country. Hence

$$\frac{P}{P^*} = \left(\frac{P_n}{P_n^*}\right)^{\alpha} \tag{1}$$

   Thus in this model the real exchange rate depends only on the internal relative price of non-traded goods.

   (a) *Let $\widehat{P}$ be the growth rate of the price level and let $\widehat{P}^*$ be the growth rate of the foreign price level. If α is constant, when will $\widehat{P} > \widehat{P}^*$?*

   brief answer Looking at (1) we can see that the only way the left-hand side can get bigger, given $\alpha > 0$ and constant, is if the price of non-traded goods rises faster at home than abroad; i.e., if $\widehat{P}_n > \widehat{P}_n^*$. Makes obvious sense, since prices in the two countries differ only by non-traded goods. Note, however, that if the $\alpha's$ differed across countries then changes in α could also cause such changes, although obviously the $\alpha's$ are bounded between 0 and 1.

   (b) *Let $\widehat{A}_T$ be productivity growth in tradable goods in the home country and let $\widehat{A}_N$ be productivity growth in the non-traded goods sector (and $\widehat{A}_T^*$, $\widehat{A}_N^*$ for the foreign country). Suppose that $\widehat{A}_T - \widehat{A}_T^* > \widehat{A}_N - \widehat{A}_N^*$. What would you expect to happen to $\widehat{P} - \widehat{P}^*$? Why?*

   brief answer It should rise. If this condition holds, it follows that $\widehat{A}_T - \widehat{A}_N > \widehat{A}_T^* - \widehat{A}_N^*$. The productivity differential at home is greater than abroad,so we should expect wages to be rising faster domestically than in the foreign country. Higher productivity growth in traded goods raises wages in the entire economy. So we should expect non-traded goods prices to rise faster domestically than in the foreign country, so therefore the domestic price level should grow faster.

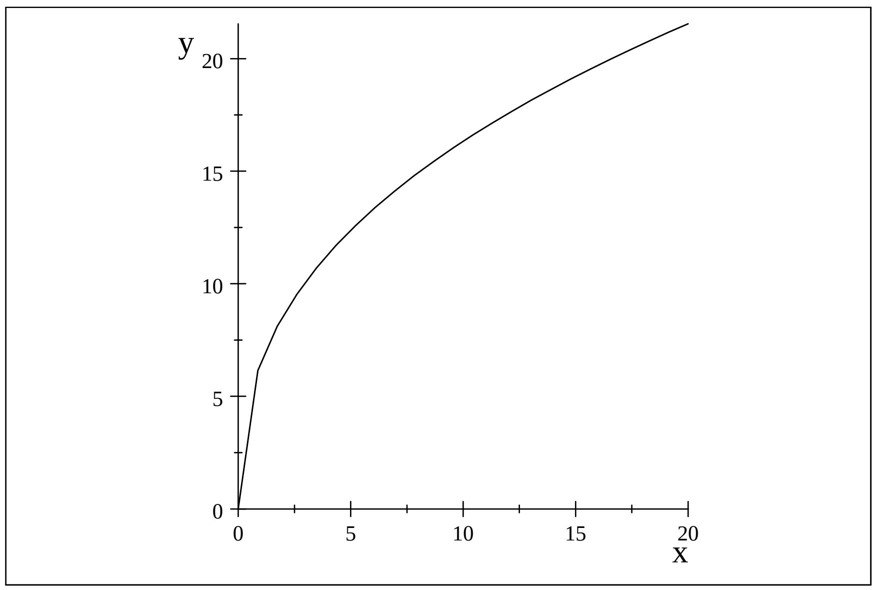
   (c) *Is the condition $\widehat{A}_T - \widehat{A}_T^* > \widehat{A}_N - \widehat{A}_N^*$ more likely to hold in richer countries or poorer countries? What then would you expect to happen to a country's real exchange rate as it gets richer?*

brief answer More likely in poorer countries that are developing. Catchup is when productivity growth in traded goods will be highest. Rich countries can only grow at the rate of technological progress, but poorer countries catch up by accumulating capital, etc. Just as Japan after WW2. In these cases their real exchange rate depreciates (for them recall that the rich country is the foreign country).

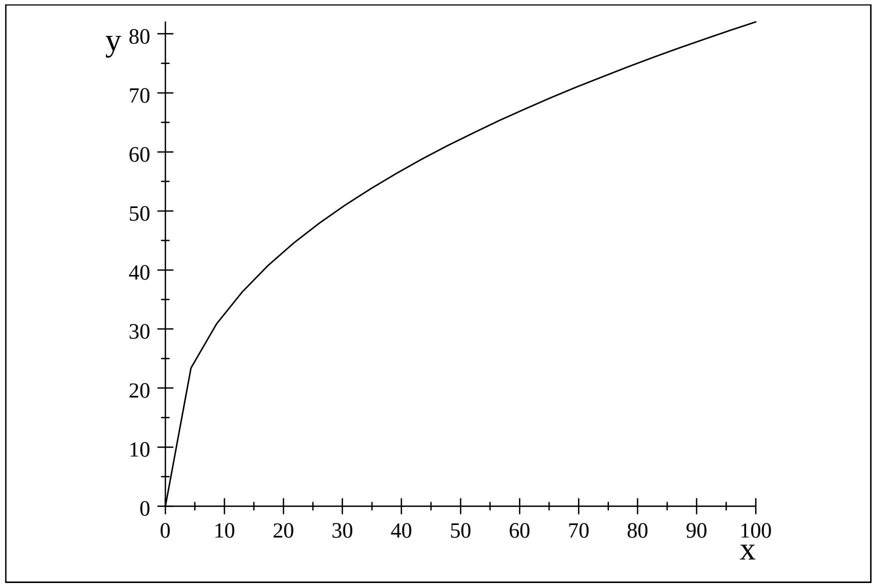
2. *Consider the dynamic model. There are two countries, US and Japan. They have identical production functions, $y = Ak^{\beta}$, where $A = 100$ in both countries, and capital's share, $\beta = 0.35$ in each country. Suppose that the savings rate $(1-\alpha)$ is .10 in the US and .20 in Japan.*

   (a) *Draw the transition curve for each country.*

   brief answer The transition equation is $k_{t+1} = (1-\alpha)(1-\beta)A_t k_t^{\beta}$. So for the US we have $k_{t+1} = (1-.9)(1-.35)(100)k_t^{.4}$and for Japan we have $k_{t+1} = (1-.8)(1-.35)(100)k_t^{.4}$



US Transition Equation: $(1-.9)(1-.35)(100)k_t^{.4}$



Transition Equation for Japan, $(1-.8)(1-.35)(100)k_t^{.4}$

(b) *Under autarky compute the steady state capital-labor ratios for the US and Japan. Which country will have higher per-capita output? Explain.*

brief answer We are working with the steady state of the model (see lecture note, page 25, equation 24). The key expression for the steady state capital-labor ratio is

$$\overline{k} = [(1-\alpha)(1-\beta)A_t]^{\frac{1}{1-\beta}}$$

We thus merely substitute and calculate. Thus for the US, we have

$$\overline{k}_{US} = [(1-.9)(1-.35)100]^{\frac{1}{1-..35}} = 16.355$$

and for Japan,

$$\overline{k}_J = [(1-.8)(1-.35)100]^{\frac{1}{1-.35}} = 51.732$$

To get per-capita output we substitute $\overline{k}$ into the production function. So for the US, we have $\overline{y}_{US} = 100(16.355)^{.4} = 265.94$, and for Japan we have $\overline{y}_J = 100(51.732)^{.35} = 397.94$

(c) *Suppose that capital markets are integrated, and assume that the US has twice the population of Japan. What happens to k in each country (i.e., what happens when integration occurs). What will the steady-state capital labor ratio be under complete integration?*

brief answer Since the rate of return is higher in the US, savings will flow from Japan to the US. This means that k will rise in the US, and will fall in Japan. We have $N^{US} = 2N^J$, so $\overline{\alpha} = \frac{1}{3}(.8) + \frac{2}{3}(.9) = 0.867$. So the world steady-state capital-labor ratio is given by:

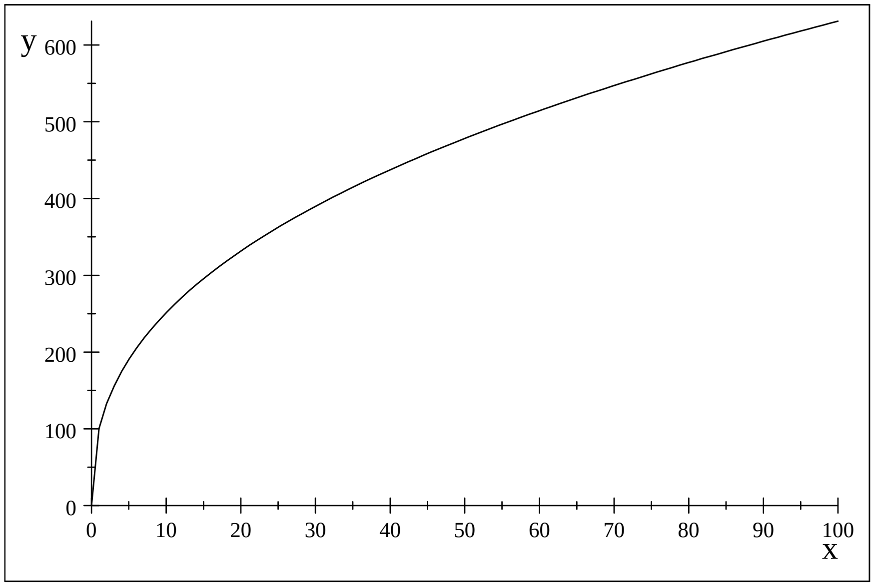
$$\overline{k}_W = [(1-.867)(1-.35)100]^{\frac{1}{1-.4}} = 27.617$$

1. *What will per-capita output be in this steady state? How does it compare to world output under autarky?*

breif answer $\overline{y}^W = 100(27.617)^{.35} = 319.46$. Under autarky, output per worker is 265.94 in the US and 397. 94 in Japan. Since the US is twice as large as Japan, world output under autarky is $\frac{1}{3}(397.94) + \frac{2}{3}(265.94) = 309.94$. So world output rises with liberalization.

2. *What will the rate of return to capital be in the new steady state [hint: find an expression for the marginal product of capital]? How does this compare to the rate of return to capital under autarky in each economy?*

   brief answer The marginal product of capital $= \beta A k^{\beta-1}$, so we have



Production Function

$$f_{\overline{k}_W} = .35(100)(27.617)^{(.35-1)} = 4.048$$

For the US and Japan we have, $f_k^{US} = .35(100)(16.355)^{(.35-1)} = 5.69$, and $f_k^J = .35(100)(51.732)^{(.35-1)} = 2.692$. You can see how much steeper the production function is at $x = 16.355$ than at $x = 51.732$.

3. *In the integrated capital markets steady state, which economy will have positive net foreign assets and which will have negative? Explain.*

   brief answer Japan will have positive NFA and the US will have negative, since Japan is investing in the US.

4. What is the level of net foreign assets for Japan and for the US? How would you figure this out?

   brief answer Notice that world output per worker is 309. 94, and the world savings rate is $(1 - .867)$, so world savings per worker is $(1 - .867)(309.94) = 41.222$. In Japan they save a higher share $(1 - .8)$, so savings per worker is $(1-.8)(309.94) = 61.988$, while in the US we save less, so savings per worker is $(1 - .9)(309.94) = 30.994$. So each Japanese citizen holds $61.988 - 41.222 = 20.766$, and each American has NFA equal to $30.994 - 41.222 = -10.228$. Since there are two US workers for every Japanese worker these offset (except for rounding error).

(d) *Suppose that productivity in the US rises permanently to* 150*. What will happen to the steady state world capital-labor ratio? Explain.*

brief answer We know that this will raise the return to investing, so the capital-labor ratio will rise. Before, we had $r_{US} = r_W = r_j = .35(100)(27.617)^{.35-1} = .35(100)(27.617)^{.35-1} = 4.048$. Now we have $r_{US} = .35(150)(19.812)^{.35-1} = 7.536$ which is greater than $r_J = .35(100)(27.617)^{.35-1} = 4.048$. So capital will flow from Japan to the US to equalize rates of return. The new equilibrium requires, $.35(100)k_J^{.35-1} = .35(150)k_{US}^{.35-1}$, or $\left(\frac{k_{us}}{k_J}\right)^{.35-1} = \frac{.35(100)}{.35(150)}$ or $\frac{k_{us}}{k_J} = \left(\frac{.35(100)}{.35(150)}\right)^{\frac{1}{..35-1}} =$ 1.8658, that is the capital-labor ratio in the US will be 1.8658 larger than in Japan. So a little more than 1/3 of the world capital stock is in Japan, the rest in the US. Thus, the world capital labor ratio, $\overline{k}^W = (1-.867)(1-.35)[\left(\frac{1}{1+1.8658}\right)100 + \left(1-\frac{1}{1+1.8658}\right)150]k^{.35}$, and note that $\left(\frac{1}{1+1.8658}\right)100 + \left(1-\frac{1}{1+1.8658}\right)150 = 132.55$, so $\overline{k}_W = [(1-.867)(1-.35)132.55]^{\frac{1}{1-.35}} = 42.604$.

1. *In the new equilibrium what will be the relative sizes of the two economies (say, in terms of their capital-labor ratios)?*

   brief answer We already showed that the US capital-labor ratio is 1.8658 times larger than Japan. We can further show that $k_w = 42.604 = (1/3)k_j + (2/3)k_{us} = (1/3)k_j + (2/3)(1.8658)k_j$, so $k_j[1/3 + (2/3)1.8658] = 42.604$, or $k_j = \frac{42.604}{1/3+(2/3)1.965} = 25.925$, so $k_{us} = 1.8658(25.925) = 48.371$. One could then show that output in the US is $y_{us} = 150(48.371)^{.35} = 583.04$, and in Japan we have $k_j = 100(25.925)^{.35} = 312.47$. We can see that productivity differences lead to large output differences, even though Japan saves more than the US.

3. *Consider a small economy so the country is a price taker in traded goods. Then we can treat foreign and domestic traded goods as a composite good, T. The country can transform capital and labor into traded and non-traded goods according to some productions possibility frontier given by its technology. The country used to receive a transfer from abroad, but now that has been withdrawn.*

   (a) *Suppose that at the initial consumption point the production possibilities frontier was linear. That is, assume that the marginal rate of transformation of traded goods into non-traded goods is constant for a segment in the neighborhood of the initial consumption point. If the transfer is withdrawn will the real exchange rate change? Explain. Use graphs.*

   brief answer It is useful section 4 of the current account lecture, part two. Under the assumption that the marginal rate of transformation is constant in the relevant range we have figure 1. The withdrawal of the transfer induces the shift from Q_0 to P_0. At P_0 there is an excess demand for traded goods. But because the PPF is linear the economy can move to point $\widetilde{C}$ without any change in relative prices. Hence, there is no change in the real exchange rate. The point is that substitution does not require any price change in this case.

   (b) *Suppose that at the initial consumption point the production possibilities frontier was very non-linear. That is, the marginal rate of transformation changes a lot as you move to the northwest or southeast. What happens to the real exchange rate when the transfer is withdrawn in this case.*

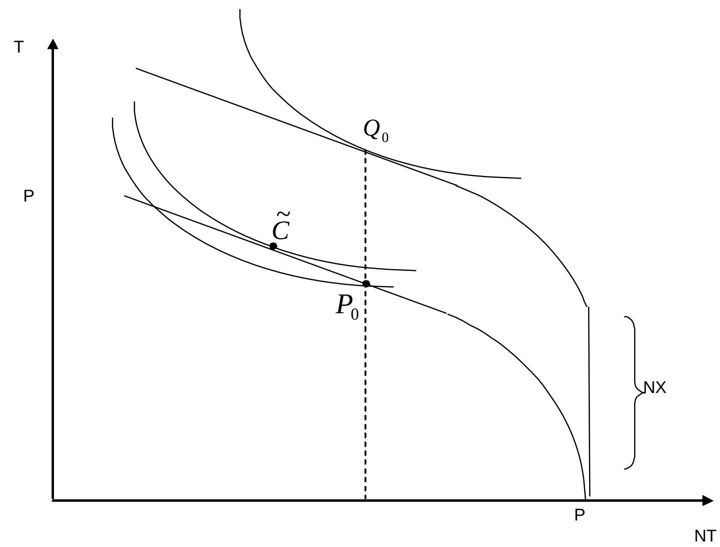


Figure 1:

brief answer This is the exact opposite case of part (a). Rather than perfect substitution there is very imperfect substitution. This means that the PPF is quite bowed, so any movement to the northwest implies a big change in the marginal rate of transformation.

(c) *Suppose that the marginal rate of transformation is more non-linear in the short-run than in the long run. What does this imply about the path of the real exchange rate when the transfer is withdrawn? Explain.*

brief answer Recall figure 19 of the current account lecture, part two. The green PPF gives the short-run possibilities. We can see that the relative price of traded goods must rise more in the short run than in the long run. This implies that in the short run the real exchange rate will overshoot its long-run equilibrium.

4. *A robust empirical fact is that price levels for services (or more generally, nontradable goods) are generally lower in poor countries. Can you explain why this might be the case?*

brief answer Over the long run, price levels are flexible and have completed their adjustments to equilibrium levels. Taking the hint from the problem set about how firms engaged in international trade may react to large and persistent cross-border price differentials, we can see why relative PPP holds more closely in the long run. Think of two economic regions, the US and Europe. Recall that PPP says that depreciation from time t to $t+1$ is given by $\frac{e_{t+1}-e_t}{e_t} = \pi_{US,t} - \pi_{E,t}$. Suppose in the short run, European inflation exceeds US inflation and that PPP does not hold. In the case of this inflation differential, then trading firms will face an incentive to substitute, where possible, US goods for European goods in their purchases on both sides of the Atlantic. If many firms over time behave in this way, then, all else equal, this should drive down the euro and drive up the dollar. This means that the dollar price of the euro will fall over time, leading the left side to become smaller and more negative. This brings the left side of

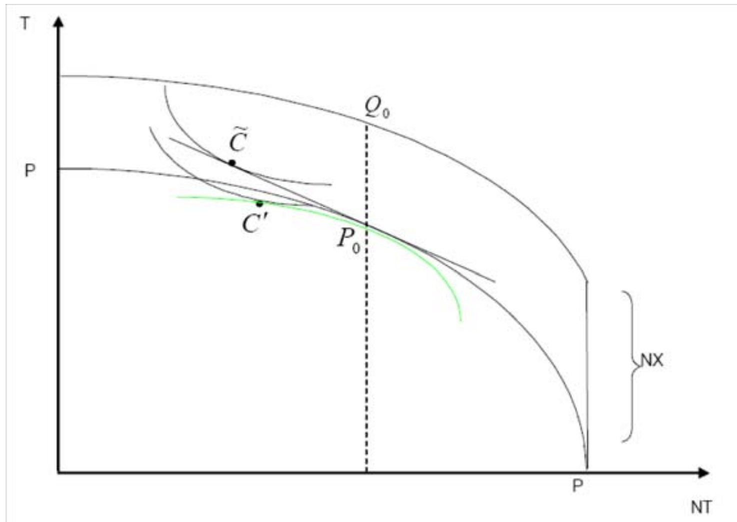


Figure 2:

the expression more into line with $\pi_{US,t} - \pi_{E,t}$. The same reasoning follows symmetrically if $\pi_{US,t} > \pi_{E,t}$. Thus, it is conceivable that purchases by international trading firms can lead to more of an alignment between inflation differentials and currency depreciation in the long run.

(a) *Suppose I told you that the price of a massage in Kenya is 3195 KES (Kenyan shillings) and the price of a massage in the US is $80. Now, go on the web and look up Kenya's actual market exchange rate (http://www.centralbank.go.ke/). Using this market exchange rate, calculate the price in dollars of a massage in Kenya. Is this price higher or lower than the US price? Is this answer consistent with the Balassa-Samuelson theory?*

brief answer The actual market exchange rate between the US and Kenya is approximately 79.6528 KES/\$. Then if the price of a massage in Kenya is 3,195 KES, its price in dollars (using the market exchange rate) is (approximately) \$40.11. Clearly, this is lower than the US price of a massage. This is largely consistent with the Balassa-Samuelson theory of international price differences. Because the US is more productive at producing tradable goods than Kenya, we expect prices of non-traded goods (and the overall price level) to be higher in the US than in Kenya.

(b) *Again using the prices given in part (b), calculate the implied PPP exchange rate between dollars and KES. Now, suppose that the Kenyan GDP is approximately 1,200 billion KES. Calculate the value of Kenyan GDP in dollars using both the market exchange rate and the PPP exchange rate you calculated. Is the value of Kenyan GDP higher or lower when using the PPP exchange rate relative to when you use the market exchange rate? Can you explain why?*

brief answer The implied PPP equilibrium exchange rate between the dollar and the Kenyan Schilling is $\frac{P_{US}}{P_K} = \frac{80}{3195} = .025$ dollars per KES (or, the direct quote for the KES is 40 KES per dollar). Then using the PPP exchange rate, we see that Kenyan GDP in dollars is 1200 billion KES times 0.025 dollars per KES = 30 billion dollars. If we use the market exchange rate instead, then Kenyan GDP in dollars is 1,200 billion KES times ($\frac{1}{79.6528}$) dollars per KES = 15.065 billion dollars. So, Kenyan GDP in dollars is higher when we use the PPP exchange rate. This makes sense because the PPP exchange is constructed so as to make currency conversions in a way that accurately represents the purchasing power of the currency. For example, at the PPP exchange rate, someone starting with \$80 in the US can pay for a massage. Converted at the PPP exchange rate, this \$80 yields exactly 3195KES, which allows one to pay for a massage in Kenya as well. In general, because prices are lower in poor countries, a poor country's GDP measured in dollars using PPP exchange rates will exceed their GDP in dollars using market exchange rates to convert currencies.

5. We can define the real exchange rate as $Q = \frac{SP^*}{P}$, where S is the dollar price of foreign currency and P^* is the foreign price level. Explain how would Q change if:

(a) *US demand for goods produced in the rest of the world declined.*

brief answer If demand falls relative to supply the price should fall. Hence, the price of US goods would fall relative to goods produced in the rest of the world, so the real exchange rate would rise. (this is just the opposite case of that considered in the lecture notes, see figure 8).

(b) *US government spending increased.*

brief answer This is mostly spend on US goods rather than foreign goods, so it creates an excess demand for US goods, so the real exchange rate should go down.

(c) *A tsunami suddenly reduced output in the rest of the world.*

brief answer Rest of the world output would be more scarce than US output, so the relative prices of US output would fall, hence the real exchange rate would rise. Of course I am assuming that demand in the rest of the world does not fall by as much as output. The key point is that our demand for foreign goods has not fallen (since our incomes did not fall) but foreign output did fall. So their output becomes more scarce. It is true that their demand for our output also falls which makes our output more abundant. Both effects make our goods less scarce than theirs, so the relative price of foreign goods must rise, so Q must rise.

(d) *A technological shock increased US output relative to world output.*

brief answer With given stocks of capital and labor US output rises. Hence, at unchanged world demand there is an excess supply of US output. Why? This positive supply shock raises US income (wealth), but not all of the increase in income is spent on domestic goods. Some will be spent on foreign goods. Hence, the increase in the demand for US goods will be less than the supply. To restore equilibrium the relative price of US goods must fall; in other words, Q must rise, and the dollar must fall in real terms. This real depreciation of the dollar (or real appreciation of the foreign currency, say the DM) means that the purchasing power of the foreign currency has increased. Thus relative productivity growth causes the real exchange rate to appreciate and the real value of the currency to depreciate.

(e) *Under what conditions would Q be invariant (unrelated) to any of the factors in parts a through c? Explain*

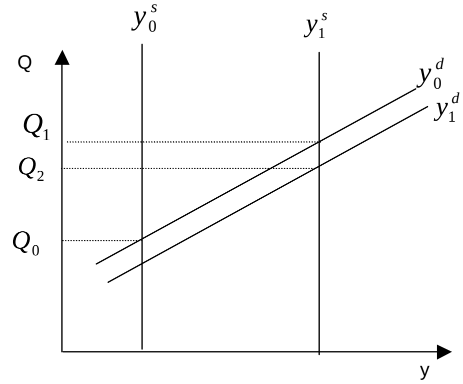


Figure 3: A change in relative supply

brief answer If there were no non-traded goods, so that PPP was true then the real exchange rate never changes. In that case goods are equivalent and arbitrage assures that they are equal. So shifts in demand for US goods do not matter because US goods and foreign goods are perfect substitutes.