Lecture Note on Exchange Rate Fluctuations

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1. Exchange Rates and Interest Parity

First we use UIPC to understand the determination of e. Recall that

$$1 + i = \frac{1}{e_t}(1 + i^*)\widetilde{e} \tag{1}$$

we can refer to the left-hand side as the dollar return ($r_\$$) and the right as the expected return on euro deposits, E_{eu}. Arbitrage keeps the two returns equal.

Now suppose we fix $\widetilde{e}$, and see how the spot rate varies with returns. It is clear from (1) that E_{eu} is negatively related to e_t. We have figure 1:

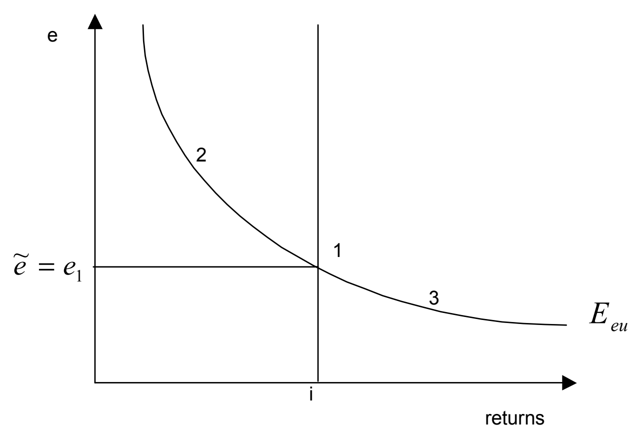

Figure 1: Determination of the Exchange Rate

Clearly E_{eu} is negatively sloped from expression (1). The equilibrium exchange rate is determined where the domestic interest rate line intersects with the E_{eu} curve.

Why is e_1 the equilibrium exchange rate?

- If $e = e_2$ then $E_{eu} < r_\$$. This causes excess supply of euros and causes the dollar to appreciate, $e_2 \rightarrow e_1$
- If $e = e_3$ then $E_{eu} > r_\$$. This causes excess supply of dollars and causes the euro to appreciate, $e_3 \rightarrow e_1$

Interest rate changes What if i changes? At the initial exchange rate E_{eu} is now less than $r_\$$. Hence, there is an excess supply of euros, the dollar will appreciate. This is evident in figure 2:

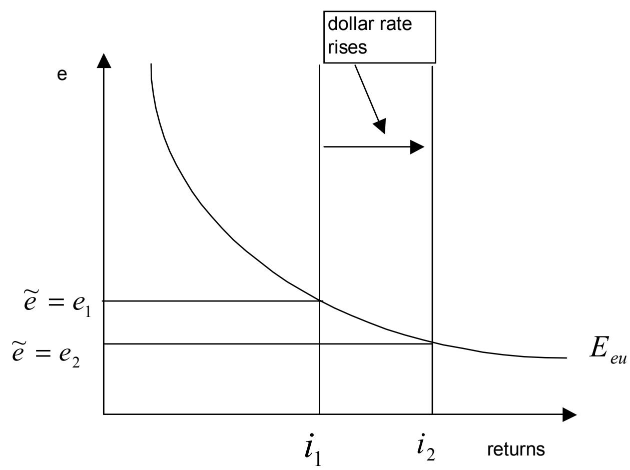

Figure 2: Impact of an increase in dollar rates on the exchange rate

A decrease in dollar interest rates would have the opposite impact on the exchange rate. If $r_\$$ falls then investors will deem foreign deposits to be superior; they will sell dollars and buy euros. The dollar would depreciate.

What happens if i^*changes? This will change the expected return on euros. Hence the E_{eu} will shift. If foreign interest rates increase the expected return on euros rises and the E_{eu} curve shifts up. At the old exchange rate the expected return on euros is higher than for dollars. So the dollar must depreciate to e_2 in figure 3.

Notice that the impact of a change in $\widetilde{e}$ can be analyzed in the same way. If people all of a sudden expect the euro to appreciate this means a higher expected return on euros at the unchanged current spot rate. There will be an excess supply of dollars. The dollar appreciates to e_2 as in figure 3.

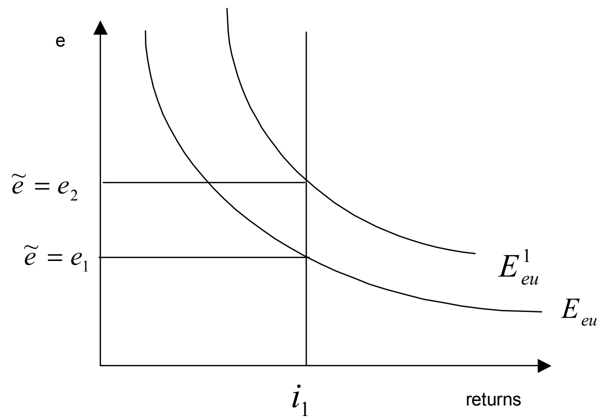

Figure 3: A Change in the Expected Return to Euros

2. Money Supply and the Exchange Rate

Now we introduce money to the equation. We need to consider money demand and supply. We assume that money demand is given by:

$$l = l(i, y) \tag{2}$$

which says that real money demand is a function of the nominal interest rate and real income. Clearly $l_1 < 0$ and $l_2 > 0$. The real money supply we will take as given. Market clearing thus implies that

$$\frac{M}{P} = l(i, y) \tag{3}$$

A rise in the nominal money supply or a decrease in the price level must cause i to fall if y is constant. In the short run it is useful to treat y as given. Hence, fluctuations in money and the price level will impact primarily on i. But we know that i impacts on the exchange rate as well.

Equation (3) determines the dollar interest rate. Hence we can use that in combination with interest parity to see how changes in the money supply impact on the exchange rate.

Suppose that the real money supply increases (M goes up or P falls). Then i falls. This means that the exchange rate must appreciate (the currency must depreciate). We have the opposite of figure 2. Indeed, we can combine the figures. In figure 4 we have the money market clearing condition in the bottom of the diagram and the exchange rate determination diagram in the top. When the real money supply increases it lowers domestic interest rate. This causes the dollar to depreciate and we move to e_2.

What about changes in the foreign money supply? Clearly we have the same forces at work. The foreign interest rate, i^*, is determined by the real supply of euros, $\frac{M_{eu}}{P_{eu}}$ and by real money demand in euroland. Thus if, say, M_{eu} were to increase, that would cause i^* to decrease. We could then trace the impact on the exchange rate as before. Given the decrease in foreign interest rates the expected return to holding euros declines, the E_{eu} curve shifts to the left (or down) as expected returns are lower. This means that at the old exchange rate people want to hold dollars, and the dollar must appreciate.[1]

[1] Notice that the change in the euro money supply caused the exchange rate to change but it left the dollar interest rate unchanged. This holds because we assumed that US real income is given. Hence, there are no impacts from the economic situation in euroland on the US money market. This is a simplification that is useful for now. Later we will examine these connections.

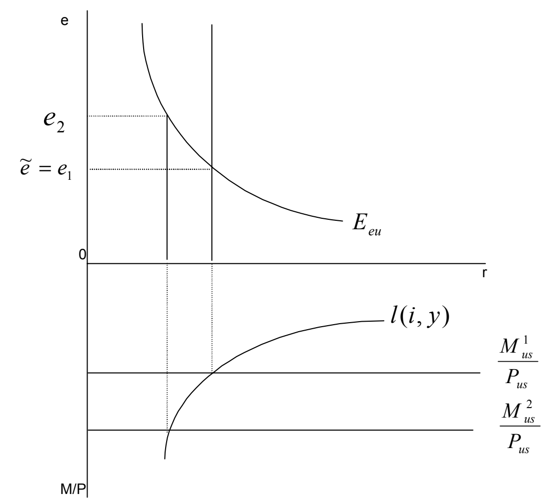

Figure 4: Impact of a money supply change on the exchange rate

2.1. Long-Run Relationship

Notice that in the long run – when i and y are at their equilibrium values, the price level adjusts to the nominal money supply. We can write this using expression (3) as:

$$P = \frac{M}{l(i, y)} \tag{4}$$

In the long run money supplies and price levels are proportional. In the short run they may not be due to rigidities, but in the long run prices adjust to keep real money supply and demand equal. When money grows exceptionally fast, as in a hyperinflation, then prices and money growth dominate everything else. Though, in such cases people use less real money, so that prices grow even faster than money.

What about exchange rates in the long run? A doubling of the money supply in the US must, in the long run, cause the dollar to depreciate against the euro by half. Otherwise, we could just change the numbers on the currency and have real effects. During the interim while prices have not yet adjusted to the higher money supply the real money supply rises and the nominal interest rate declines. But once prices have adjusted the real money supply returns to where it started. Because there is now more dollars but the same number of euro, the exchange rate between the two must increase so that the dollar price of euro is now higher. In terms of figure 4 the dollar interest rate returns to where it was, but because the price level rises there must be an increase in the expected return to the euro. Because E_{eu} shifts to the right, the full equilibrium involves a higher value of e. This is not a new result. Recall Hume's outline of the specie-flow mechanism:

> "Suppose four-fifths of all the money in Great Britain to be annihilated in one night...what would be the consequence? Must not the price of all labour and commodities sink in proportion, and everything be sold as cheap as they were in those ages? What nation could then dispute with us in any foreign market, or pretend to navigate or to sell manufactures at the same price, which to us would afford sufficient profit? In how little time, therefore, must this bring back the money which we had lost, and raise us to the level of all the neighbouring nations? Where, after we have arrived, we immediately lose the advantage of the cheapness of labour and commodities; and the farther flowing in of money is stopped by our fullness and repletion." Hume, *Of Money*.

But what about the short-turn dynamics, and the adjustment to the new equilibrium?

2.1.1. Overshooting

Now we examine the dynamic adjustment to the full equilibrium after a permanent increase in the money supply. The analysis begins as with figure 4. In the short-run – call this the *impact* effect – prices are still fixed. After impact prices adjust. Once prices have fully adjusted we are in the new equilibrium. As before, we assume that real output is constant. This is a simplification to make life easy. It is not necessary for our results.

Start with figure 5 where the rise in the money supply to M_1 raises the real money supply and causes i to fall to i_1. With expectations unchanged the exchange rate would rise to e_2. But this ignores the impact on expectations about the price level. If agents are rational they will realize that prices in the US must rise relative to Europe. And they must also understand that this will cause the exchange rate to rise in the future. Hence, they expect that the dollar will depreciate. This causes the expected return on the euro to rise, so E_{eu} shifts to the right and the exchange rate rises further to e_3. The impact effect of the monetary expansion is thus to lower the interest rate to i_1 and to cause the exchange rate to rise to e_3.

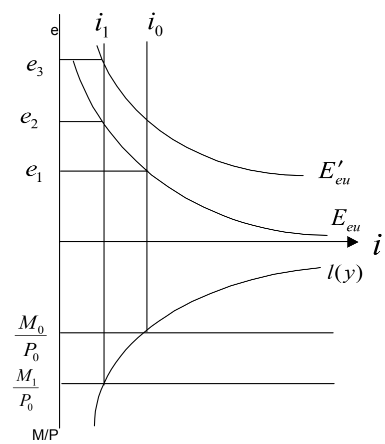

Figure 5: Impact Effect of a Permanent Increase in M

After impact, if nothing else changes, prices must eventually rise. In the new equilibrium P rises to P_1 and the real money supply returns to its initial value; that is, $\frac{M_1}{P_1} = \frac{M_0}{P_0}$. What happens as the price level increases? Because output is unchanged, the decrease in the real money supply must cause the interest rate in the US to rise back to its initial level. We assume no further change in exchange rate expectations, so the rise in US interest rates must cause the exchange rate to decrease. This follows because the return on holding dollars is rising compared with euros.

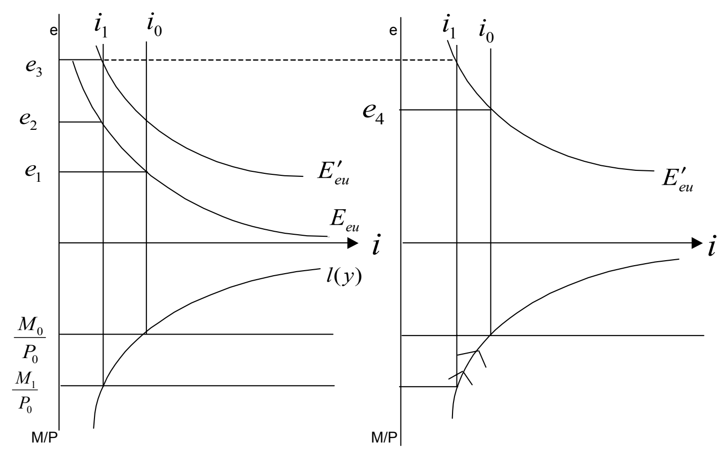

Figure 6: Dynamic Adjustment to the Rise in M

We can see this in figure 6. We can see that as the price level increases the real money supply contracts and the interest rate in the US increase. This causes the exchange rate to decrease to e_4. We observe that the long-run impact on the exchange rate is *smaller* than the impact effect. This is referred to as *overshooting*. The reason why the exchange rate overshoots is that the price level is rigid in the short run. If prices were as flexible as exchange rates then we would immediately jump from e_1 to e_4 with no effect on the interest rate. But because of the differential flexibility of prices and exchange rates the latter must adjust more in the short run. Overshooting is a response of asset markets with rational agents. If there were no overshooting arbitrage possibilities would occur. To see this, ask what would happen if the exchange rate rose only to e_2 or e_4 at impact. At these exchange rates the expected return of the euro would exceed that of the dollar. This would cause people to sell dollars and buy euros until the exchange rate rose to e_3.

An Alternative Analysis of the Same Model An alternative apparatus to see overshooting can be developed. This is useful for it helps to focus ideas. As overshooting is an important factor in asset markets it is worth the effort. Again we focus on money market equilibrium and on interest parity conditions. We also continue to assume that output is constant and that prices adjust more slowly than exchange rates. And we use the fact that in the long run exchange rates are proportional to prices.

Begin with the money market equilibrium condition, expression (3), $\frac{M}{P} = l(i, y)$. Now from the uncovered interest parity condition we know that

$$i = i^* + \frac{\widehat{e}_{t+1} - e_t}{e_t} \equiv i^* + \delta \tag{5}$$

where $\widehat{e}_{t+1}$ is the expected exchange rate next period. Now we can use the UIPC to substitute for i in the money market equilibrium condition:

$$\frac{M}{P} = l(i^* + \delta, y) \tag{6}$$

Notice that in long-run equilibrium $\delta = 0$, because exchange rates are no longer expected to change. But in the short run, when $\delta \neq 0$, this will enable us to see a relationship between P and e. Moreover, for the US we can treat i^* as an exogenous variable: it should be relatively unaffected by our policies.

We need one more assumption to complete the analysis – rational expectations. We assume that agents understand how the economy works, so that there forecasts of future exchange rates coincide with what the model predicts. Hence, if I let $\overline{e}$ be the equilibrium exchange

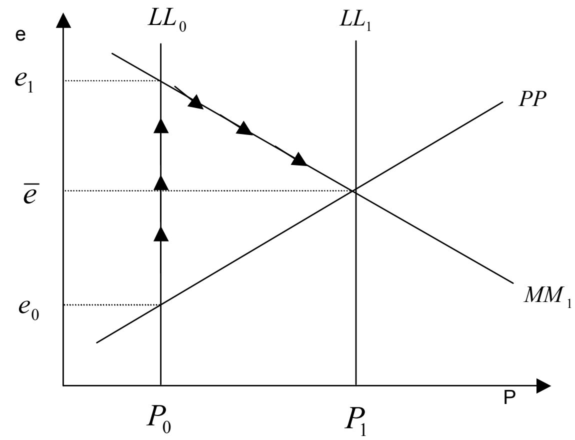

Figure 7: Overshooting

rate, we can assume that $\widehat{e}_{t+1} = \overline{e}$. This gives us an idea of where the economy is headed.

Now consider the relationship between P and e that satisfy expression (6). In the long-run $\delta = 0$, so we have $\frac{M}{P} = l(i^*, y)$; hence, any value of e is consistent with money market equilibrium. We have the LL curve in figure 7. That is what money market equilibrium tells us, but we also know from purchasing power parity condition that the exchange rate is proportional to the price level.[2] We label this relationship PP in figure 7. The intersection of these two curves gives us the long-run equilibrium exchange rate.

What about the short run when $\delta \neq 0$? Notice that if $\delta > 0$ ($e_t < \overline{e}$) this means that money demand decreases. At a given money supply the price level must rise to maintain money market equilibrium. If $\delta < 0$ ($e_t > \overline{e}$) then money demand is higher than in equilibrium so the

[2]Recall that PPP is the assumption that the real exchange rate is constant, so that movements in spot exchange rate reflect movements in relative price levels. Hence, if P^* is the price in euroland, then $e = \frac{P}{P^*}$. Thus with foreign prices taken as given, e is proportional to P.

price level must be lower. Hence, we observe a negative relationship between the spot exchange rate and the price level that maintains money market equilibrium. Of course we could have just as clearly spoken of P not being at its long-run equilibrium value and ask what must happen to δ. We label the combinations of e_t and P that satisfy (6) in the short run, MM. It should be obvious that in long-run equilibrium LL, MM, and PP intersect at the same combination of e and P.

Now put the parts together. We start at the initial price level and exchange rate (e_o, P_0). Now suppose that the money supply is increased. In the long-run prices will rise proportionately, to P_1. The LL schedule shifts to LL_1. Given purchasing power parity the exchange rate in the new equilibrium will be higher – the dollar depreciates due to the monetary expansion. The new long-run equilibrium exchange rate is given by $\overline{e}$. But prices do not, in fact, adjust instantaneously. When the money supply increases prices are given. The only way to satisfy money market clearing is with decreased money demand. Given that output is fixed this can only occur if people expect exchange rate depreciation $(\delta < 0)$. But $e_0 < \overline{e}$, so this is clearly not satisfied. The exchange rate must rise immediately, in fact it must overshoot the long-run equilibrium increase so that $\delta < 0$. This is evident from the MM schedule. The exchange rate appreciated immediately to e_1, and then as prices increase it depreciates to the long run value. We get the familiar (by now) overshooting shape.

Why does the exchange rate overshoot? This follows, once again, from the assumptions about adjustment speed. Notice that a monetary expansion means that at unchanged prices there is an excess supply of money. To restore money market equilibrium the opportunity cost of holding domestic money must fall so that money demand can increase. The only way this can happen is if agents expect that $\delta < 0$ so that $i^* + \delta$ can fall. But the only way that agents can rationally expect the exchange rate to *depreciate* is if the exchange rate immediately jumps above the new full equilibrium value.

Another way to think about overshooting is to think about the adjustment in prices. At impact the price level is unchanged but over time income will rise to P_1. That means that real money demand will decrease as $P \longrightarrow P_1$. Because we are considering a one-time change in the money stock, and because that occurs before P rises, this would create an excess demand for money along the adjustment path. To keep the money market in equilibrium as P rises the decrease in money demand must be offset. Hence, the interest rate must be falling along the adjustment path to the new equilibrium. But the money stock is now larger. The only way to get to full equilibrium is if the interest rate first rises sufficiently at impact so that agents will expect depreciation.

We can see that arbitrage opportunities would arise if e did not overshoot. In the full equilibrium we know that $\delta = 0$ and that $i = i^*$. Because $\overline{e} > e_0$, no overshooting would imply that the exchange rate would appreciate – and the currency depreciate – on the path to the new equilibrium. But if the currency depreciates in value and domestic interest rates equal foreign interest rates why would anyone hold domestic currency? They will dump dollars and buy foreign currency. This will make the exchange rate increase. When will the dumping of domestic currency end? Until agents expect sufficient currency appreciation to make them once again willing to hold domestic currency.

The overshooting model thus offers an explanation of why asset prices respond rapidly to new information.

Of course in practice the economy is subject to many shocks, so asset prices fluctuate in the kind of saw-tooth pattern that is characteristic of these markets.

2.1.2. Anticipated Policies

An interesting feature of the overshooting model is that *anticipated* policies have immediate effects. Consider an announcement that the money stock will be increased next period. This will cause an appreciation of the exchange rate and a rise in income in the new full equilibrium. At impact, however, prices have not yet risen. So asset prices feel the full brunt of the change in anticipations. Notice that the expected exchange rate increases as in the case of an unexpected increase in the money stock. So the LL curve shifts to the right, and the current exchange rate overshoots the full equilibrium adjustment.

What is interesting about this case is that the exchange rate increases *before* the money supply rises. The market *anticipates* the effect. Notice further that with the exchange rate appreciated prices will start to rise even before the money supply increases. This makes sense because the higher exchange rate will increase the demand for our goods relative to our competitors and this will spark aggregate demand. In the short run output may rise in accord with Hume, but as we have held this fixed, the impact will be on prices. The reason why prices rises in our model is the real exchange rate appreciation which makes the economy more competitive. When the money stock does finally increase there is no discernible effect on the exchange rate, because that has already been *absorbed* in the price.[3] Only if there is a further change in the money supply that was *not expected* will there be a change in the exchange rate, for that would be news.

2.1.3. Speculative Bubbles

The role of expectations makes it apparent how a bubble can arise in asset markets. A bubble is a situation where asset prices move because they are expected to move. That is not the same thing as anticipated policies (discussed in section 2.1.2.). In that case the asset price is following fundamentals; the fact that the change has not yet occurred yet does not mean that expectations are not based on fundamentals. In a bubble, on the other hand, the price moves away from fundamentals based solely on expectations of further movements. Expectations become *self-confirming*. You may not think this is possible, but stock market analysts think this way all the time. They argue about market momentum and technical analysis. This is the

[3] Unless the money stock increases in a manner that was unanticipated – for example by a smaller amount. Then there would be a new adjustment to the "new" news.

argument that prices move independent of fundamentals, or expectations thereof.

Can bubbles occur? Our discussion of attitudes towards uncertainty suggests how they can. People over-react to news. But it is a leap from saying that they can happen to the statement that observed episodes were bubbles. It is worth some consideration here.

Economists have discussed several episodes of speculative bubbles: Tulipmania,[4] South Sea Bubble, the dollar in 1985. Now we all talk about the telecoms bubble. Consider, for example, figure 8 which shows the US price-earnings ratio relative to profits. One can clearly see that prices rose relative to profits dramatically in the 1990's, totally out of line with previous experience.[5]

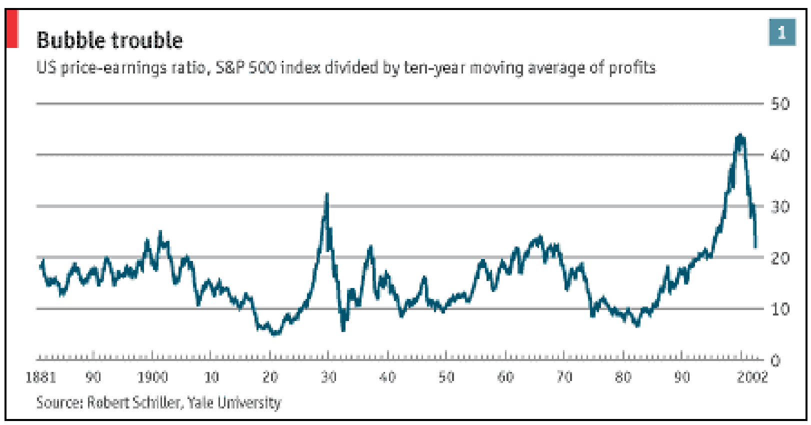

Figure 8: Price Earnings Ratio

Despite the apparent evidence, Peter Garber has argued that many so-called bubble episodes have not been properly interpreted. In Tulip-

[4]Though Garber has argued that Tulipmania was not in fact a speculative bubble.

[5]This is taken to be evidence of a bubble. But it could reflect a structural change. Perhaps a change in tax treatment, or a shift in the regulatory regime that reduces future downturns. Or the new economy.

mania, for example, a Semper August bulb sold for 2000 guilders in 1625, an amount of gold worth $16,000 at $400/oz.[6] In 1636 prices rose speculation ensued and then prices collapsed. So goes the story. According to the chronicles, in 1637 prices collapsed and bulbs could not be sold for more than 10% of their value. Stories of sailors eating a $10,000 bulb is used as evidence of the mania, or that of four oxen and 1000 pounds of cheese being exchanged for one bulb. Explaining the tulip price puzzle actually requires that 3 entirely different components be dealt with.

1. High Prices Paid For Some "Rare" Tulips Bulbs
2. Price Patterns For "Rare" Bulbs (e.g. Semper Augustus, 1623-1739)
3. Price Patterns For "Common" Bulbs (i.e. January-February 1637)

As to the rare bulbs, these are luxury items. Newly introduced their prices may be very high. The "luxury good" explanation of tulip prices is based on tulips being scarce, on there being only limited quantities of a very much desired good/product. Garber (1990) points out that at the time, tulips were subject to a mosaic virus whose effect ("breaking") was to produce remarkable patterns on the flower (today this tulip mosaic virus no longer exists). Tulips can propagate either through seeds or through outgrowths or buds on the mother bulb. The valued mosaic pattern, however, cannot be reproduced through

[6] The price of one special, rare type of tulip bulb called Semper Augustus was 1000 guilders in 1623, 1200 guilders in 1624, 2000 guilders in 1625, and 5500 guilders in 1637 (equal to current (1990) US$ 50.000 in gold). Another bulb was sold in February 1637 for 6700 guilders. On these price levels one single tulip bulb could cost as much as a house on Amsterdam's smartest canal, including coach and garden. The average annual income at the time was only 150 guilders. After the "crash" prices are said to have fallen to less than 10 percent of their peak values and by 1739 prices had fallen to 1/200 of the peak price. Clearly, such price movements are in line with a bubble hypothesis: start low, reach high, end low.

the much more easy seed propagation, but can only be reproduced by cultivating the buds from the original bulb. The size of the original bulb determines the number of outgrowths and therefore its value. The outgrowths are not ready to flower themselves until after a period of approximately 3 years. Every common bulb could "break" at some unknown time and with unknown patterns. Tulips changed from one season to another.

As for the second point here, it must be noted that the prices quoted for were futures contracts on bulbs. These contracts were not legal – the government would not enforce them – but if the trades were profitable nobody cared. There were no margin requirements and no marking to market. So little wealth was needed to speculate here. But when prices really exploded they could be bought out for 10%. So if the price fell by 10%, say from 100 to 90, by paying 10 you could buy out of the contract. This is not a 90% fall in the price, but a 10% fall.

Notice, in fact, that this is really not a futures contract at all, but an option. The strike price is 100 you pay 10 for it. Suppose the expected price is 50. If there is some chance that the price may rise – volatility – you may purchase this option. A price of 10 may not be too high for this bet. But if this is an option, then 100 surely is not the actual price people were paying, *or even expecting*. It was the strike price of an out of the money option.

Most of the stories about expensive tulips turn out to be false,[7]. As for the common bulbs Garber offers a plausible explanation: From 1635-1637 a bubonic plague epidemic ravaged the Netherlands, killing 10-30 percent of the population in cities such as Amsterdam, Leiden, Haarlem. It appears reasonable that common people regarded tulip speculation in the same way as they do today a million-dollar lottery: an opportunity to become rich instantly and gain the opportunity to escape the drudgery of daily live (i.e. death from the bubonic plague). A "rational gambling" theory suggests that even uncertain projects

[7] Rhetorical arguments in pamphlets became facts in subsequent re-tellings. There was no such sailor. After all, what type of merchant would leave a $10,000 tulip sitting on a table?

with negative expected monetary values can be positively priced when one positive outcome is sufficiently extreme to be on an entirely different utility curve (effectively creating a quasi-convex utility function). The lottery was made possible by the market system: no margin requirement, no marking to market.

The basic argument here is that one should be very careful about the history. It seems quite logical that a tulip as expensive as a house must be the sign of hysteria. What else could it be? But there may be a fundamental explanation.

A modern manifestation of Tulipmania could be derived from a report on the *Economist* (January 22, 2000, p.57), which reported that a resident of Cuba paid $4,300 for a 25-year-old American car. The car was in deplorable condition, had windows that would not close and doors that wouldn't open in a normal fashion. In the USA such a car would be worth less than its weight in scrap metal. The average wage in Cuba equals around $10 a month. It seems that Cubans are quite happy to spend many years' salary on an obviously worthless commodity. Perhaps in twenty years time some clever historian will report that Cubans of the 1990's must be considered to have been quite mad and irrational. Obviously, there is a clear fundamental explanation that has a lot to do with the shortage of autos.

Garber also argues that fundamentals may have driven the prices.[8] He notes that even to this day new, rare bulbs fetch high prices, and then they decline over time. Similarly Garber argues that the South Sea Bubble and the Mississippi Company of John Law may also have been driven by fundamentals. Both of the latter companies sold equity to buy up low-grade government debt. They then invested in the colonies of the state and in various government granted monopoly activities. They were state-related finance companies and government

[8] In fact, he found evidence of a fundamental shock: In France, it became fashionable for women to wear quantities of fresh tulips at the top of their gowns, and men competed to present the most exotic flowers. In fact, there is evidence that a single flower of a particular broken tulip was sold for 1000 guilders in Paris – but this was final demand, it was purchased as a consumption good.

officials were closely tied to them. Perhaps there were expected returns to such colonial investments that justified the high prices. The collapses, it seems arose from fundamental shocks too: the King sold his shares in the Mississippi company, for example. With government support missing expectations ought to change.[9]

What Garber does is argue that there are potential fundamental explanations that would justify high valuations. If earnings projections turned out correct prices would be justified and there is no bubble. But if expectations are rational why don't we ever observe periods when above book value prices are followed by huge earning growth? Why are investors *always* unlucky?

Supposing that there is evidence of a bubble in asset prices we want to ask how can such self-confirming equilibria occur? One answer, of course, is given in the terminology often used: *mania, fad, panic*, etc. And perhaps we should just take a psychological view. But given that these are financial markets and people have a lot of money invested here we want a bit more. It turns out that we can explain a *rational bubble.*

Think of an asset price, e_t, and suppose that fundamentals suggest that the exchange rate should be e^* for all t. At some time t_0 the exchange rate jumps to e_0 as in figure 9. Now suppose further that agents expect that the value of the dollar will increase at the rate $1 + r$ in each period. For example, suppose $b_t = b_0(1 + r)^t$ for arbitrary b_0. In other words, the price of domestic currency is increasing exponentially. Why are agents willing to pay increasing prices for the dollar? The price is growing each period due to expectations of future price growth. The expected capital gains are self-fulfilling – this is a rational bubble. The price of the asset bears no relation to fundamentals. It is the anticipation of never-ending appreciation of the price of the asset that keeps the bubble from bursting. The anticipation of ever-increasing prices is self-fulfilling and it satisfies the

[9] A fad explanation also suffers from the fact that the explosion in the South Sea company's shares from £350 to £750 occurs just as the Mississippi Company's shares are collapsing by 50%.

arbitrage condition: the capital gain on holding dollars compensates for the alternative returns. Of course, for this to occur the price will have to grow forever. If everyone knew that at period $T+j$ that the bubble would burst ($e \rightarrow e^*$ or $b_t \rightarrow 0$) then no one would pay the bubble price at $T+j-1$. The bubble unravels.

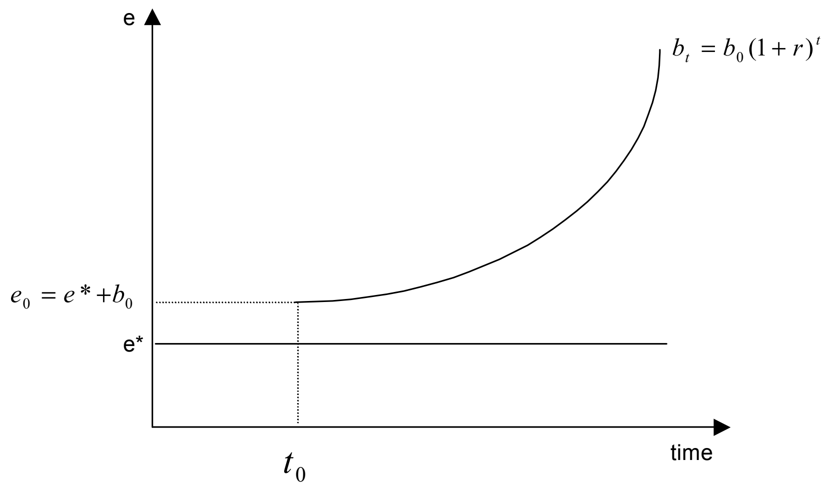

Figure 9: A bubble in the exchange rate

People are not idiots. They know that the price cannot literally increase forever. Bubbles can still arise, however, as long as expected price growth compensates for the capital loss that occurs when the bubble bursts. Suppose then that in each period agents believe that the probability that the bubble will not burst is q. Then we have:

$$b_{t+1} = \begin{cases} \frac{(1+r)b_t}{q} + \varepsilon_{t+1} & \text{with probability } q \\ \varepsilon_{t+1} & \text{with probability } 1-q \end{cases} \tag{7}$$

where ε_{t+1} is a white noise error, with mean 0. If the bubble follows this path it is rational. The reason is that the expected value of the bubble in period $T+1$ is exactly $b_{t+1.}$ To see this, note that

$E_t[b_{t+1}] = q\frac{(1+r)b_t}{q} + (1-q)(0) = (1+r)b_t$, which is our initial expression for the bubble path. The bubble continues with probability q, and it bursts with probability $1-q$. If it bursts, it returns in expected value to zero. Notice that this means that the bubble must grow faster (since $q < 1$) than it would in the previous example, because investors must be compensated for the risk of the bubble bursting. It is precisely this sort of reasoning that people used to think about the value of the dollar prior to the Plaza meeting of 1985.

Indeed, if the bubble is rational you can back out the market's expectation of it bursting. At any t you know the actual price and the interest rate. If you know the fundamental price (e.g., e_0) then you can use the fact that if the bubble has not burst $b_{t+1} = \frac{(1+r)b_t}{q}$ so that

$$q = \frac{(1+r)b_t}{b_{t+1}}.$$

This notion of a rational bubble is used frequently in analysis of asset markets. The path of the dollar, in particular, was thought by many to be on a bubble path in the mid- 80's. The reason was that fundamentals such as the balance of trade, interest differentials, and other determinants of exchange rates did not seem to explain the movements of the dollar. It seemed higher than could be explained by fundamentals.

2.1.4. Why this is important

Understanding bubbles is important for many reasons, but for our purposes the most important one is that many financial crises seem to be associated with bubbles in asset prices. These seem to follow financial liberalization, and is especially critical in emerging markets.

One explanation of these phenomena points to the fact that much investment in asset prices is financed by debt. If the ultimate source of funds cannot observe the riskiness of the investment then there is a classic risk-shifting situation. The borrower has an incentive to take excessive risks, as limited liability limits the downside risk. The agent captures most of the gain of successful investments but losses are limited. This causes the bubble. When credit is cheap the financing of the bubble is greater.

3. Absolute and Relative PPP

We now turn to the monetary approach to exchange rate determination. This is a good way to start thinking about exchange rate detemination. It is is a long run theory. It is relatively straightforward to explain and makes strong predictions about long-run movements. It is less effective about short run, but we will come to that.

The law of one price suggests that the spot exchange rate is determined by relative price levels:

$$e_t = \frac{P_{US}}{P_E} \tag{8}$$

Expression (8) is a theory of exchange rate determination – *purchasing power parity* – based on the assumption that all goods are tradeable.[10] Hence, it assumes that real exchange rates are constant. It is not a bad assumption for the long run, but it may be problematic for the short run. If each country produced one and the same good, and if transport costs and national prejudices did not exist, then arbitrage would clearly bring about (8). Of course countries produce many goods, and not all are tradeable. It is nonetheless worthwhile to see its implications.

Recall the assumption that the money market clears. This implies:

$$P_{US} = \frac{M^s_{US}}{l(i_{US}, y_{US})} \tag{9}$$

[10] Suppose that the basket of goods that were produced in the US and Germany were identical, and that all goods were tradeable. In that case, net of transportation costs we would have the law of one price: arbitrage would insure that the dollar prices of the various goods would be identical across countries. This yields a theory of exchange rate determination known as PPP.

and it also implies that for Europe:

$$P_E = \frac{M_E^s}{l(i_E, y_E)} \tag{10}$$

Price levels depend on money demand and money supply. But from (8) we know that the exchange rate depends on the ratio of price levels. Hence the exchange rate is going to depend on what happens to prices in the US relative to Europe, and this depends on what happens to the money supply in the US relative to Europe. You can also see that if output rises permanently in Europe, this will raise money demand in Europe, and all things constant, lower the price level in Europe. Hence the exchange rate will appreciate.

It thus follows that any changes in national price levels results in a movement of the exchange rate. PPP thus determines the exchange rate by the movements in relative price levels. If US inflation is higher than foreign inflation the exchange rate will appreciate and the dollar will depreciate relative to the foreign currency. It will take more dollars to purchase a euro. This is intuitive: the nominal exchange rate is the relative price of currencies, and inflation is the measure of the decrease in purchasing power of a currency. If the dollar is losing purchasing power faster than a euro, then the euro should gain in value relative to the dollar.

This can be seen more clearly if we look at movements in the exchange rate and growth rates of the price level, inflation (π). From (8) we can write:

$$\frac{e_t}{e_{t-1}} = \frac{\frac{P_{US,t}}{P_{E,t}}}{\frac{P_{US,t-1}}{P_{E,t-1}}} = \frac{\frac{P_{US,t}}{P_{US,t-1}}}{\frac{P_{E,t}}{P_{E,t-1}}} \tag{11}$$

Now define inflation as $\pi_t = \frac{P_{US,t}}{P_{US,t-1}} - 1$. So we can write (11) as:

$$\frac{e_t - e_{t-1}}{e_{t-1}} = \frac{1+\pi_{us}}{1+\pi_E} - 1 \tag{12}$$

$$= \frac{1+\pi_{us}}{1+\pi_E} - \frac{1+\pi_E}{1+\pi_E} \tag{13}$$

$$= \frac{\pi_{US} - \pi_E}{1+\pi_E} \tag{14}$$

Now it is clear that $\pi_{US} - \pi_E = (\pi_{US} - \pi_E)(1 + \pi_E - \pi_E)$, so I can write (14) as:

$$\frac{(1+\pi_E)(\pi_{US} - \pi_E)}{1+\pi_E} - \frac{\pi_E\,(\pi_{US} - \pi_E)}{1+\pi_E} = (\pi_{US} - \pi_E) - \frac{\pi_E\,(\pi_{US} - \pi_E)}{1+\pi_E} \tag{15}$$

But if inflation rates are rather low the difference between them is likely to be low, and the product of this difference and the inflation rate is likely to be even lower. Hence, for low inflation rates the last term on the right hand side of 15 $\rightarrow 0$, which means that we have the approximation:

$$\frac{e_t - e_{t-1}}{e_{t-1}} = \pi_{US} - \pi_E \tag{16}$$

which is called *relative purchasing power parity.*

Expression (16) says that the percentage change in the nominal exchange rate is equal to the difference between the inflation rates in the domestic and the foreign country.[11] When price levels are changing very rapidly these movements can dwarf all other factors, and then PPP provides a rather effective theory of exchange rate movements. A great advantage of this expression is that it holds even if absolute PPP does not.

[11] Recall that $\frac{X_t - X_{t-1}}{X_{t-1}} = \frac{X_t}{X_{t-1}} - 1$ is the percentage change in X, and thus $1 + g = \frac{X_t}{X_{t-1}}$, where g is the percentage growth rate. Now if we take logs of this expression, for small g, it follws that $log(1+g) \approx g \approx x_t - x_{t-1} \equiv \Delta x_t$.

Now recall that from UIPC we have:

$$\frac{e^e_{t+1} - e_t}{e_t} = i_{US} - i_E \tag{17}$$

So if expected inflation differences correspond with actual inflation differences, it follows that the interest differential will be equal to the difference in expected inflation rates. Moreover, as market participants understand expression (16) it follows that expected inflation will be equal to the growth rate of the exchange rate. Hence,

$$i_{US} - i_E = \pi^e_{US} - \pi^e_E \tag{18}$$

where π^e_{US} is the expected US inflation rate. Thus expression (18) says that the interest differential will be equal to the difference in expected inflation rates.

Notice what expression (18) implies. If agents expect higher US inflation relative to Europe it follows that US interest rates must rise relative to European rates. Hence, according to expression (18) the real return on US assets relative to Europe will be unchanged. This is called the *Fisher effect*. The *Fisher effect* is usually written as $r = i - \pi^e$. Movements in expected inflation leave real interest rates unchanged. But this is also what is implied by expression (18).

Suppose then that money growth in the US rises relative to Europe. In the long run we would expect that inflation would rise by an equal amount. And so would expected inflation. Hence we would expect the interest rate in the US to rise relative to Europe. This also implies that the exchange rate must appreciate at the same rate as the interest differential. In the long run output and real returns are unchanged. The only effect of the rise in money growth is on the nominal quantities. Of course, we know in the short run there will be impacts. But we can also see that in the long run only nominal quantities are effected.

A simple example of this theory is provided by the Big Mac index. The Big Mac is essentially the same good in every country. Hence, we can compare the dollar price of Big Mac's across countries. Where the currency appears over-valued we should expect the exchange rate to appreciate, and vice versa. This does surprisingly well though it is not perfect. Notice that even with the Big Mac, however, price differences persist. Not even all the Big Mac costs are really tradeable.

3.0.5. Problems with PPP

Even relative PPP does not do all that well in explaining exchange rates. This is evident in the comparison of German and US price levels and the exchange rate (figure 15-3 in the book). It is evident that relative price indices explained a lot more prior to 1973 than since. Although the price levels do seem to explain the trend, the deviations keep getting larger, and fairly long lasting. This is a general result: relative *PPP* was much better prior to 1973 than since. This suggests that the move to floating exchange rates has something to do with it. An anchor is eliminated. We shall return to this idea.

The fact that PPP does not hold suggests that we look at the real exchange rate. Now the real exchange rate is the relative price of goods in each country, we can write it as:

$$Q = \frac{eP^*}{P} \tag{19}$$

where P^* is the price level in the foreign country. The real exchange rate is just the relative prices of goods (the nominal exchange rate is the relative price of currencies). As is evident in figure 10 this has moved around a lot since 1973. This volatility in the real exchange rate requires some explanation.

There are several reasons why *PPP* does not hold in the short run. Notice, that *PPP* is a theory of exchange rate determination based on goods flows. It is tied to trade (it is not the only theory of this kind), and it ignores capital flows. Exchange rates can also fluctuate because of expectations of future changes, though even these must be based on something. We have talked about current accounts. Of course, the need to finance deficits can lead to different rates of inflation, and so

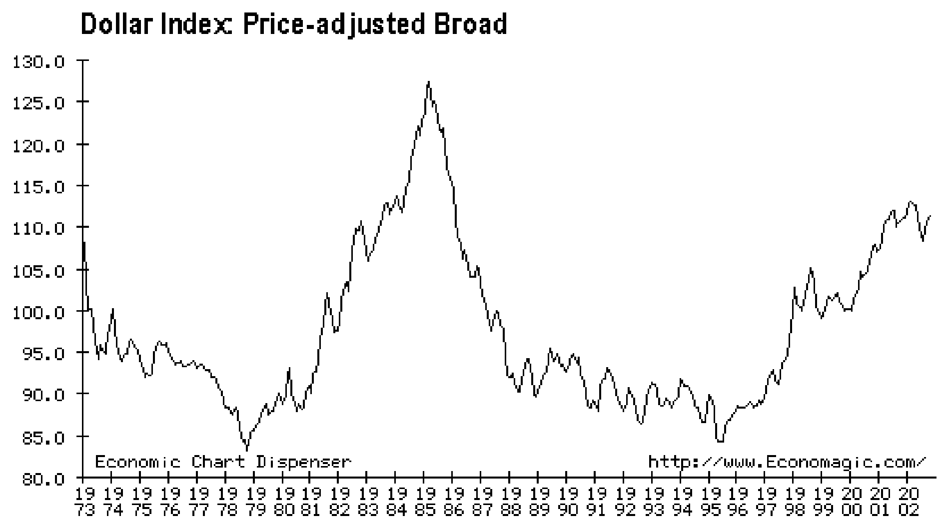

Figure 10: Real Exchange Rate of the Dollar

back to *PPP*. So it is not trivial to dismiss it. Here are some important issues.

- First, tariffs and transportation costs create a band in which prices can fluctuate before arbitrage becomes profitable.

- Second, permanent shifts in the terms of trade can cause the real exchange rate to change, if countries differ in the composition of output. An oil shock (positive) will have a different effect on an energy producer and a producer of energy-intensive products. The latter country will experience a relative decline in the world demand for its goods, so its currency will experience a real depreciation.

- Third, if prices are sticky in the short run the law of one price, by definition, does not hold. Then it follows that movements in nominal exchange rates will also affect the real exchange rate.

This may hold in the short run, but over longer periods of time prices do adjust and *PPP* is more likely to hold.

- Fourth, the presence of *non-traded* goods, is probably the most important factor. Think of haircuts versus wheat. Even if traded goods are identical across countries and obey the law of one price, non-traded goods do not. Shifts in the relative price of traded and non-traded goods can cause PPP to fail. This is rather easy to see.

- Fifth, capital flows can lead to speculative movements in the exchange rate in the short run. Of course we still would need to understand why speculation was not stabilizing.

Notice that different baskets of goods is a common thread here. Price levels measure different things in different countries.

What causes changes in Q? Two specific causes are worth discussing here. It is useful, however, to think about a very simple model of real exchange rate determination. Since the real exchange rate is the relative price of foreign goods relative to domestic, a rise in the real exchange rate must lead to an increase in the demand for domestic goods. For simplicity let the supply be independent of Q (nothing is lost by this).[12] Then we have figure 11 and the initial equilibrium with demand given by y_0^d and the real exchange rate, Q_0.

1. *A change in world relative demand for US goods.* Suppose that preferences shifted so that total world spending on US goods increased. This could be due to shifts in private demand towards US goods, or an increase in US government spending which is concentrated on US goods. At current exchange rates this would cause an excess demand for US goods. To restore equilibrium

[12] If we considered the impact of Q on aggregate supply we would conclude that it is negatively sloped: a decline in the real exchange rate is an increase in the relative price of domestic goods, so production should increase.

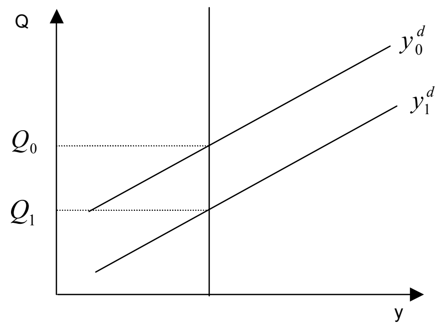

Figure 11: An Increase in the Demand for Domestic Goods

the relative price of US goods must rise relative to foreign goods; hence, Q must fall, and the dollar has appreciated in real terms. In other words, the purchasing power of the dollar has increased relative to foreign goods. This is evident in figure 11. The increase in the demand for US goods causes the demand curve to shift to the right and the real exchange rate falls to Q_0.

2. *A change in relative output supply.* Suppose that there is a relative technological shock that increases the efficiency of US output relative to foreign output. With given stocks of capital and labor US output rises. Hence, at unchanged world demand there is an excess supply of US output. Why? This positive supply shock raises US income (wealth), but not all of the increase in income is spent on domestic goods. Some will be spent on foreign goods. Hence, the increase in the demand for US goods will be less than the supply. To restore equilibrium the relative price of US goods must fall; in other words, Q must rise, and the dollar must fall in real terms. This real depreciation of the dollar (or real appreciation of the foreign currency, say the DM) means that the purchasing power of the foreign currency has increased. Thus relative productivity growth causes the real exchange rate

to appreciate and the real value of the currency to depreciate. In this case the situation is given by figure 12, where the supply has shifted to the right. In the absence of any induced impact on aggregate demand the real exchange rate would surely appreciate from Q_0 to Q_1. But given that US income rises, and some of this is spent on US goods, the demand for home output rises – but not as much as supply. So we end up at Q_2.

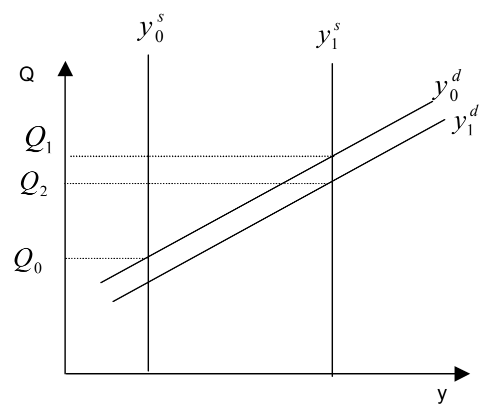

Figure 12: A change in relative supply

Real Exchange Rate and the Balassa-Samuelson Effect Our concern is with the fact that different countries have different baskets of goods, in particular, different baskets of traded and non-traded goods. Let us write the price index of the domestic country as $P = P_n^{\alpha} P_t^{1-\alpha}$, where P_t is the price of traded goods, and α is the share of non-traded goods in the domestic price index. Then we can write the real exchange rate as:

$$
\begin{aligned}
Q &= e\left[\frac{P_n^{*\alpha^*}P_t^{*(1-\alpha^*)}}{P_n^{\alpha}P_t^{1-\alpha}}\right] \\
&= e\left[\frac{\left(\frac{P_n^*}{P_t^*}\right)^{\alpha^*}P_t^*}{\left(\frac{P_n}{P_t}\right)^{\alpha}P_t}\right] \\
&= e\left(\frac{P_t^*}{P_t}\right)\left[\frac{\left(\frac{P_n^*}{P_t^*}\right)^{\alpha^*}}{\left(\frac{P_n}{P_t}\right)^{\alpha}}\right] \qquad (20)
\end{aligned}
$$

but if we assume that PPP holds for traded goods, it follows that $e\left(\frac{P_t^*}{P_t}\right) = 1$, so

$$
Q = \left[\frac{\left(\frac{P_n^*}{P_t^*}\right)^{\alpha^*}}{\left(\frac{P_n}{P_t}\right)^{\alpha}}\right] \qquad (21)
$$

Expression (21) tells us that the real exchange rate will change if the *relative* price of non-traded goods changes in either the domestic or foreign country. Why might these relative prices change? Changes in demand and production structure as a country grows is one big reason. As an economy modernizes and grows the price of non-traded goods rises relative to traded goods. One big reason is that wages tend to rise.

Notice that if take logs of both sides of (21), and use lower-case to represent the log of a variable, we obtain:

$$
q = \alpha^*(p_n^* - p_t^*) - \alpha(p_n - p_t)
$$

taking first differences, we obtain:

$$
\Delta q = \alpha^*(\Delta p_n^* - \Delta p_t^*) - \alpha(\Delta p_n - \Delta p_t) \qquad (22)
$$

which says that the real exchange rate will depreciate if the relative price of non-traded goods (i.e., relative to traded goods) rises in the domestic country or decreases in the foreign country.

Now suppose that $\alpha = \alpha^*$, then we can write (22) as $\Delta q = \alpha(\Delta p_t - \Delta p_n) - \alpha(\Delta p_t^* - \Delta p_n^*)$. Now we expect that the price of tradeables will grow at the same rate in the two countries, hence $\Delta p_t = \Delta p_t^*$, which implies that

$$\Delta q = \alpha(\Delta p_n^* - \Delta p_n) \tag{23}$$

This expression says that the change in the real exchange rate is driven by the difference in the growth rates of the price of non-traded goods. Now if we think that non-traded goods prices grow faster in faster growing economies, then these economies will have lower real exchange rates, or real currency appreciation. For example, we might assume that the inflation rate of non-traded goods prices is related to productivity growth in a country. For simplicity assume that they are equal so $\Delta p_n = \Delta a$, then we could have

$$\Delta q = \alpha(\Delta a^* - \Delta a) \tag{24}$$

and the real exchange rate movements are driven by productivity differentials.

There is good reason to think that such changes do occur. The Balassa-Samuelson effect focuses on the impact of differential economic growth. The main idea is that productivity differences are greater in traded goods than non-traded goods. It is argued that economic growth is associated with increased productivity in *traded* goods, so that they fall relative to the price of *non-traded* goods. In a country with higher productivity wages will be higher. So non-traded goods will be more expensive in the high productivity country. You can think of traded goods as more tangible than non-traded goods (like haircuts). In countries that grow rapidly (or liberalize for that matter) non-traded goods will rise relative to traded goods. If productivity growth is higher in the US than in the rest of the world, our price level will rise relative to the rest of the world. This implies that q would fall.

This could also happen if non-traded goods are superior in consumer's demand functions. Either way, this relative price change causes the real exchange rate to decrease; in other words, the real value of the domestic country appreciates. This is, of course, what happened in Japan as rapid growth lead to very rapid increases in the price of non-traded goods (such as golf club memberships).

The fact that q is lower in countries that grow faster may also explain why the price level tends to be higher in richer countries when measured in common currency units. Americans often wonder how people in LDCs can live on incomes of \$500 a year. Of course, there is real poverty, but it is also the case that because of non-traded goods, conversion at exchange rates gives an incorrect impression. That is, the differences in nominal incomes do not measure the true differences in purchasing power. This is because purchasing power of a currency differs depending on the shares of traded and non-traded goods. To see this, recall that from the definition of the real exchange rate, $Q_t = \frac{e_t P_t^*}{P_t}$, we can write:

$$q_t = e_t + p_t^* - p_t$$

hence,

$$p_t = e_t + p_t^* - q_t \tag{25}$$

which implies that countries with lower q will have higher price levels compared with prices elsewhere, since the foreign price level measured in units of domestic currency is just $e_t + p_t^*$. If productivity growth is rapid in the US relative to the foreign country, our price level will be higher, when measured in common currency units.

One way to think about this is simply that PPP values exchange rates according to the relative price of traded goods. But in LDC's the price of non-traded goods is lower. When these are included, the price level in the advanced country is higher. That is essentially what is implied by (25).

3.0.6. Balassa-Samuelson and a Monetary Union

We can derive an interesting relationship for later study. First we note that in any country changes in real wages occur only with changes in productivity.[13] Hence, we can write:

$$\dot{p_i} = \dot{w_i} - \dot{\lambda_i} \tag{26}$$

where a dot above a variable is a rate of change.[14] Hence the rate of inflation in country i is the difference between nominal wage growth and productivity growth. If we had a second country we would have:

$$\dot{p_g} = \dot{w_g} - \dot{\lambda_g} \tag{27}$$

Now if relative PPP holds then the inflation rates of the two countries would determine the movement of the nominal exchange rate, as in expression (16):

$$\dot{e} = \dot{p_i} - \dot{p_g} \tag{28}$$

Now if there is a monetary union, $\dot{e} = 0$, so $\dot{p_i} = \dot{p_g}$. This implies that wage differentials are related to productivity differentials:

$$\dot{w_g} - \dot{w_i} = \dot{\lambda_g} - \dot{\lambda_i} \tag{29}$$

But this all assumes PPP. Suppose there are non-traded goods, and suppose that the price of non-traded goods depends only on wage costs. Then we can define inflation as:

$$\dot{p_g^c} = \alpha \dot{p_g} + (1 - \alpha)\dot{w_g} \tag{30}$$

[13]Suppose that output is given by $y = L^{\alpha}K^{1-\alpha}$. Profit maximization implies that $\frac{w}{p} = \frac{\partial y}{\partial L} = \alpha L^{\alpha-1}K^{1-\alpha}$. Now $\frac{\alpha y}{L} = \frac{\alpha[L^{\alpha}K^{1-\alpha}]}{L} = \alpha L^{\alpha-1}K^{1-\alpha}$, so $\frac{\partial y}{\partial L} = \frac{w}{p} = \frac{\alpha y}{L}$. If we take logs and derivatives, and noting that α is a constant, we get $\dot{w} - \dot{p} = \dot{q}$.

[14]This is just the continuous time version of of the model. If we let x be the log of some variable, then $\Delta x = x_t - x_{t-1}$ is the percent growth rate. Now if the length of the time period gets really small we have $\dot{x}$.

where α is the weight of traded goods in total inflation.[15] This follows because we know that wages in traded and non-traded goods sectors are equalized. Similarly for country i:

$$\dot{p}_i^c = \alpha \dot{p}_i + (1 - \alpha)\dot{w}_i \tag{31}$$

When i and g (Ireland and Germany) join a monetary union then $\dot{p}_g = \dot{p}_i$, so (31) implies:

$$\dot{p}_g^c - \dot{p}_i^c = (1 - \alpha)[\dot{w}_g - \dot{w}_i]. \tag{32}$$

Now we know from (29) that wage differentials depend on productivity differentials, so:

$$\dot{p}_g^c - \dot{p}_i^c = (1 - \alpha)[\dot{\lambda}_g - \dot{\lambda}_i]. \tag{33}$$

Now what is the implication of (33)? It says that in a monetary union if Ireland has higher productivity growth than Germany it will have a higher inflation rate rate than Germany. The reason is that it will have higher growth in non-traded goods prices. In a sense, this should not be worrisome – it is an equilibrium adjustment to differences in economic fundamentals in the two countries.

This result is causing some problems for Germany today, however. The reason is that it overall inflation in the monetary union is low, and Germany has the lowest productivity growth. Hence, its inflation rate must be lower than average. Suppose the union was just Germany and Ireland, and that they are equal in size. Then if you know that Union inflation is 2% and Irish inflation is 4% then German inflation is -2%. The point is that the common monetary policy of the union causes especially low inflation in the low growth country. Now when the monetary union was formed people thought, I think, that it would be the Italy's, with big deficits and high structural inflation that would have the low productivity growth. Not Germany. This is causing some consternation because Germany does not want deflation. Of course, if it could implement reforms that would increase productivity growth these problems would go away.

[15]Notice that I changed the notation. I am now using α for the share of traded goods, rather than non-traded goods.

Rising Yen The Balassa-Samuelson effect may also help explain the rising yen. In nominal terms the yen has strengthened greatly since WW2. Between 1950 and 1999 the dollar lost two-thirds of its value against the yen. Notice that much of this happened when there was a fixed exchange rate between the dollar and yen. What it reflects is higher Japanese inflation prior to 1973 than in the US. But subsequent to that US inflation was higher than in Japan. Yet, the movements in the exchange rate cannot be due to differences in inflation alone, however, as US inflation has not been that much higher than Japanese (though it has been and continues).

From the data we see that in real terms the dollar has depreciated against the yen for more than forty years. Why? Differential productivity growth in traded and non-traded goods. The relative price of non-traded goods in Japan has increased much more than in the US. Think of golf club memberships.

3.0.7. Interest Differentials and the Real Exchange Rate

According to (18) interest differentials are a function of differences in expected inflation. But we derived this relationship under the assumption of *PPP*. This is equivalent to assuming that the real exchange rate is constant. Yet, we know it is not. So how is the theory modified.

Recall that the real exchange rate is defined as $Q_t = \frac{e_t P_t^E}{P_t^{US}}$. We are interested in an expression for the *expected* growth rate of the real exchange rate, $\frac{Q_t^e - Q_{t-1}}{Q_{t-1}}$. Suppose that inflation was expected to equal in the US and euroland. Then clearly we would have $\frac{Q_t^e - Q_{t-1}}{Q_{t-1}} = \frac{e_t^e - e_{t-1}}{e_{t-1}}$. Of course expected inflation rates are not equal, however, so how is the expression altered? Suppose that the exchange rate was not expected to change. Then clearly we would have $\frac{Q_t^e - Q_{t-1}}{Q_{t-1}} = \pi_E^e - \pi_{US}^e$. If US inflation is higher than in euroland the real exchange rate depreciates.

Put these two factors together and it is clear that:[16]

$$\frac{Q_t^e - Q_{t-1}}{Q_{t-1}} = \frac{e_t^e - e_{t-1}}{e_{t-1}} - (\pi_{US}^e - \pi_E^e). \tag{34}$$

The expression is intuitive.

Now recall the interest parity condition, $\frac{e_t^e - e_{t-1}}{e_{t-1}} = i_{US} - i_E$. Using this to replace the expected rate of depreciation in (34), we obtain:

$$i_{US} - i_E = \frac{Q_t^e - Q_{t-1}}{Q_{t-1}} + (\pi_{US}^e - \pi_E^e) \tag{35}$$

which implies that interest differentials depend on expected movements in the real exchange rate in addition to differences in expected inflation. When the real exchange rate is not expected to change we have the same expression as before. But in general, nominal interest differentials are explained by movements in the real exchange rate as well. Of course you can think of the latter as capturing all the reasons for exchange rate movements other than differential price levels.

This now lets us derive an expression for *real interest parity*. While nominal returns are those that are the actual components of exchanges, it is real returns, or rather *expected* real returns that govern decision-making. That is, you may earn a nominal interest rate of 10% from an asset, but whether you choose to hold it or not depends on how the real return relates to the opportunity cost of the funds. So an investor

[16] To prove this take logs of the expression for the real exchange rate expression:

$$\log Q_t = \log e_t + \log P_t^E - \log P_t^{US}$$

Now differentiate both sides with respect to time and we obtain:

$$\left(\frac{1}{Q_t}\right)\frac{dQ_t}{dt} = \left(\frac{1}{e_t}\right)\frac{de_t}{dt} + \left(\frac{1}{P_t^E}\right)\frac{dP_t^E}{dt} - \left(\frac{1}{P_t^{US}}\right)\frac{dP_t^{US}}{dt}.$$

But these expressions are just the (continuous time) growth rates of the real and nominal exchange rate and of the price levels in euroland and the US, respectively. So as the period shrinks, we obtain the expression in the text.

thinking of where to hold her wealth must consider the expected real returns.

We know that nominal interest differentials are related to expected changes in the real exchange rate and expected inflation. We obtain the real interest parity condition by first using the Fisher equation, $i_t = r_t + \pi^e$. It follows that the expected real rate of interest in the US, $r^e_{US,t} = i_{US,t} - \pi^e_{US,t}$, and likewise for Euroland, $r^e_{E,t} = i_{E,t} - \pi^e_{E,t}$. Using these in expression (35) yields the real interest parity condition:

$$r^e_{US,t} - r^e_{E,t} = \frac{Q^e_t - Q_{t-1}}{Q_{t-1}} \tag{36}$$

which says that expected real interest rate differentials are equal to expected changes in the real exchange rate.

Why should (36) hold? Suppose that people expect the real exchange rate to depreciate (the real value of the euro to appreciate), say because Euroland productivity growth in tradeables is expected to be higher than in Euroland non-tradeables, and also higher than in the US. Thus people expect the real value of the dollar to depreciate relative to the euro. To compensate for this the real return on dollar assets must exceed those of Euroland assets.

Does this mean that there are profit opportunities that are not being arbitraged away? No. The differences in the real rates do not reflect different returns on the *same asset*. It reflects different returns on two bundles of goods. The absence of arbitrage opportunities is guaranteed by interest parity, since any investor that compares relative returns has a unique consumption basket. When I compare the rate of return on holding dollars or euros, the real return is computed by subtracting my expected rate of inflation, whatever consumption basket is relevant for me. But the expected real differential on the left-hand side of (36) is comparing two expected inflation rates that reflect two *different* consumption baskets. Notice that if all agents were identical PPP would hold and we would not have real interest differentials. But because people in different countries consume different baskets of goods, so there is no way for them to arbitrage away any difference.