

## Midterm Exam II: Answer Sheet

1. (40%) Suppose that the trade balance is given by  $T = \bar{T} - mY + \phi q$ , where  $\bar{T}$  is the autonomous trade balance,  $m$  is the marginal propensity to import,  $q$  is the real exchange rate, and  $\phi > 0$ . Further suppose that domestic absorption is given by  $A = \bar{A} + aY - br$ , where  $\bar{A}$  is autonomous expenditure,  $r$  is the real interest rate, and  $a, b > 0$ . Equilibrium in the goods market requires that  $Y = A + T$ , or  $Y - A = T$ .

- (a) Explain the meaning of the equilibrium condition and show this graphically in  $T - Y$  space (i.e., the trade balance measured on the vertical axis and income on the horizontal).

brief answer The equilibrium condition is that aggregate demand equals output, or the excess of national savings over investment is equal to the trade balance. Graphically, we have figure 1. We might also note for later that

$$NS - I = Y - A = Y - \bar{A} + aY + br = Y(1 - a) - \bar{A} + br$$

So the  $NS - I$  curve has a vertical intercept at  $-(\bar{A} - br)$ , and its slope is given by  $1 - a$ .

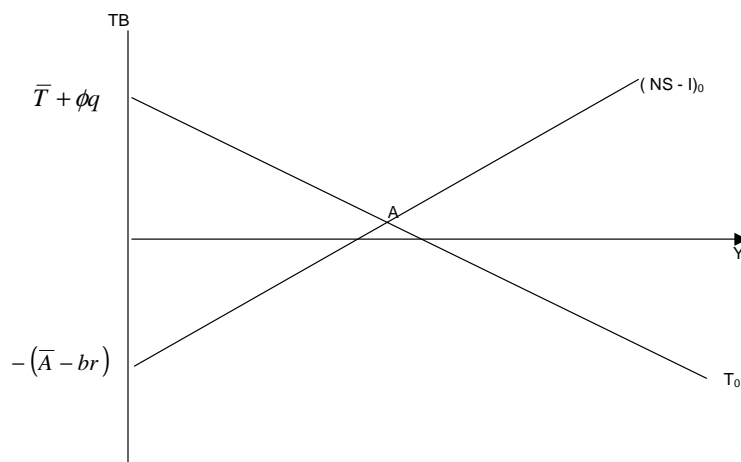


Figure 1: Goods Market Equilibrium

- (b) Show graphically and explain what happens to the equilibrium level of income and to the trade balance if:

1. there is an increase in  $\bar{A}$ .

brief answer If  $\bar{A}$  goes domestic absorption rises, so  $NS - I$  shifts to the right in figure 1. This means that the equilibrium level of rises and the trade balance deteriorates. Greater domestic absorptions means more spending at every level of income.

2. *there is a decrease in  $r$ .*

brief answer A decrease in the interest rate raises investment at every level of income, so  $NS - I$  shifts up. This causes the equilibrium level of income to rise and the trade balance to fall. [Notice that this is just a movement along the IS curve. As you move along the IS curve to the southeast interest-sensitive spending increases and net exports go down. The composition of aggregate demand changes, but every point on IS is goods market equilibrium.]

3. *a rise in foreign income.*

brief answer A rise in foreign income means an increase in  $\bar{T}$ , as we export more at every level of income. So the  $T$  curve shifts upwards in figure 1. This means that the trade balance improves and equilibrium income goes up. This is why we always want our trading partners to expand their economies. It reduces our problems!

(c) *Suppose that initially the trade balance is in deficit. If this causes the value of the domestic currency to fall, what will happen to the trade balance? Explain.*

brief answer We know that a depreciation of the currency means an increase in the real exchange rate,  $q$ , because prices are fixed in this model. If  $q$  increases the  $T$  curve shifts upwards and the trade balance improves (as in part b.iii).

While I'm At It I was originally going to ask here how your answer would be affected by the openness of the economy. That is, would the required change in  $q$  be larger or smaller if the economy were more open to trade (measured by the ratio of exports plus imports to GDP). It is an important question. How should we think about it? A rise in  $q$  alters relative prices and makes us more competitive. Suppose that the trade deficit is \$500 billion dollars initially, with exports of \$600 billion and imports of \$1.1 trillion while GDP is \$10 trillion (this is roughly close to current levels). Now suppose that the elasticity of demand for exports with respect to  $q$  is 1 and that the elasticity of demand for imports with respect to  $q$  is 1 (this is just for simplicity, it makes the calculations easy, since then exports and imports respond proportionally to the change in the real exchange rate. Then  $\eta^* + \eta = 2$  which is greater than unity so the Marshall-Lerner condition is satisfied, and we know that  $\phi > 0$  as the question assumed. We want to know how much  $q$  has to rise so that the trade deficit improves by \$500 billion. That means we want to find the change in  $q$  so that  $600\Delta q + 1100\Delta q = 500$  (I have dropped nine zeroes for simplicity).<sup>1</sup> So we have  $\Delta q(1700) = 500$ , or  $\Delta q = \frac{500}{1700} \approx .294$ . Then exports will rise to  $(1.294)600 = 776.4$ , and imports will fall to  $(1 - .294)1100 = 776.6$ , or balance trade (ignoring rounding errors). Now suppose that we are much less open an economy, but GDP and the deficit is the same. Let exports be 100 and imports be 600. Then  $100 - 600 = -500$ , as before. Assuming that the  $\eta = \eta^* = 1$  as before, then we have  $\Delta q(100 + 600) = 500$  or  $\Delta q = \frac{500}{700} = .714$ . So exports rise

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<sup>1</sup>Where did this come from? We start with the initial trade balance,  $600 - 1100 = -500$ . We want to find the change in the real exchange rate that will make this equal to zero, i.e.,

$$600 + \Delta q 600 - [1100 - \Delta q 1100] = 0$$

Add initial exports and imports to both sides and you have the expression in the text.

to  $1.714(100) = 171.4$  and imports fall to  $(1 - .714)600 = 171.6$ , again balanced trade. So with a less open economy the required change in the real exchange rate is larger. This makes sense. We need a proportionately bigger change in the real exchange rate since we need exports to rise more and imports less to reduce the trade balance by the same \$500 billion when we are less open. Notice that I have not taken into account the impact of the effect of the subsequent income change on the trade balance, but that will not change the result only the magnitudes of the calculations. You can think of this calculation as how much  $q$  must rise to shift the  $T$  curve upwards by \$500 billion.<sup>2</sup>

- (d) *Suppose that  $a$  is very large (approaches unity). What does this mean economically? What impact does this have on the size of the fall in the value of the domestic currency required to move the trade balance into surplus? Explain.*

**brief answer** If  $a$  is very large  $1 - a$  (the slope of the  $NS - I$  curve) goes to zero – it is very flat. Economically, this means that net national savings ( $NS - I$ ) is not very sensitive to income. The reason is that savings is unresponsive to income because we spend domestically almost every cent we earn when income rises. The situation is then given in figure 2. There is a current account deficit at point  $A$ . Even if  $q$  rises and  $T$  shifts upward the trade deficit remains. Income will rise, but most of that will be spend on domestic goods. So no there is very little switching of expenditure towards net exports. The real exchange rate would have to rise *a lot* and that means that dollar would have to fall a lot to restore the trade balance.

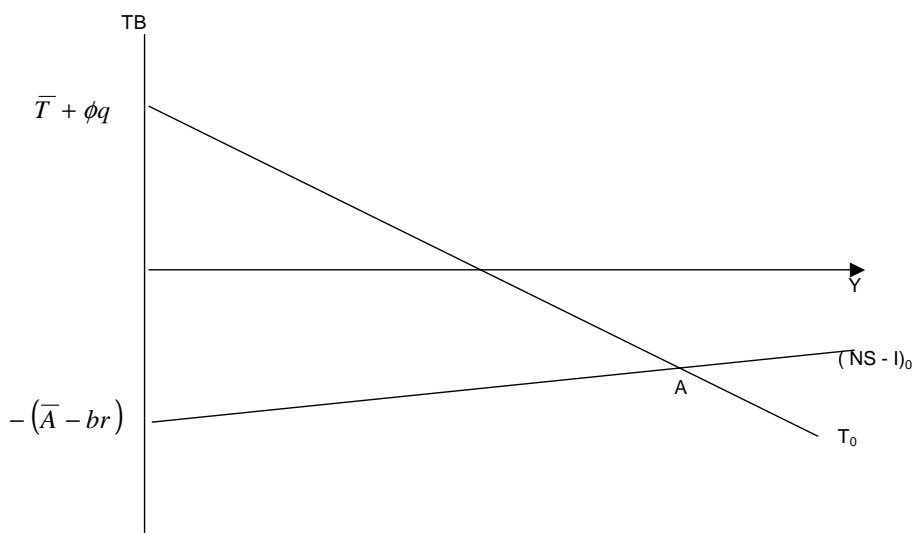


Figure 2: Savings Insensitive to Income

- (e) *If changes in the value of the domestic currency are unlikely to restore the trade balance to surplus, what other policies might do it? Explain. Use graphs where possible.*

**brief answer** We need to reduce domestic absorption so net exports can increase. Policies that might achieve this would be a rise in taxes or a fall in government spending

<sup>2</sup>For an analysis taking into account the induced changes in income, see the appendix.

– either one reduces  $\bar{A}$  and so shift  $NS - I$  upwards. A rise in interest rates would also have that effect. This is not surprising – a rise in  $r$  would increase savings and reduce investment (as we learned in the inter-temporal choice model, and this reduces the current account deficit). What you must observe from this, however, is that these policies would result in income falling, so they are more painful than just letting the dollar depreciate (although that would cause some inflation and that is not pretty either). Notice that getting foreigners to buy more of our goods (for example, complaining about China, would not help – even if they did by more US goods, that just means a rise in  $\bar{T}$ , and we have already seen in part (d) that this is ineffective when  $1 - a$  is very small.

2. (30%) Let  $\pi_{US}^e$  be expected inflation in the US, and  $\pi_E^e$  be expected inflation in Euroland. Suppose that purchasing power parity holds.

(a) If  $\pi_E^e$  rises relative to  $\pi_{US}^e$  what happens to the expected depreciation of the currency,  $\delta \equiv \frac{s_{t+1}^e - s_t}{s_t}$  (where  $s$  is the number of dollars per euro)? What happens to the nominal interest differential?

brief answer If we expect higher Euro inflation then the price level in Euroland will be higher in the future relative to ours. Hence, from *PPP* we know that the dollar must appreciate or  $s$  must fall. Hence holding dollars will result in a capital gain relative to euros, and the interest differential  $i_{US} - i_E$  must fall. This is just the interest-parity condition. Under *PPP* it is movements in price levels that explain exchange rate movements.

(b) What does this imply about the real interest differential,  $r_{US} - r_E$ ? What is the real interest differential equal to in this case?

brief answer The real interest differential will not change, it is still equal to zero. Under *PPP* we know that  $\frac{s_t - s_{t-1}}{s_{t-1}} = \pi_{US} - \pi_E$ , i.e., relative purchasing power parity.<sup>3</sup> The Fisher relation says that the real interest rate is equal to the nominal rate minus expected inflation:  $r_{US} = i_{US} - \pi_{US}^e$ . Uncovered interest parity tells us that  $i_{US} - i_E =$

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<sup>3</sup>PPP implies that  $P_{US} = sP_E$ . If we take logs of both sides and differentiate with respect to time we immediately get the relative purchasing power parity condition. Alternatively, we can show this by noting that  $\frac{s_t}{s_{t-1}} = \frac{\frac{P_{US,t}}{P_{E,t}}}{\frac{P_{US,t-1}}{P_{E,t-1}}}$ . Now define inflation as  $\pi_t = \frac{P_{US,t}}{P_{US,t-1}} - 1$ . So we can write this as:

$$\begin{aligned} \frac{s_t - s_{t-1}}{s_{t-1}} &= \frac{1 + \pi_{us}}{1 + \pi_E} - 1 = \frac{1 + \pi_{us}}{1 + \pi_E} - \frac{1 + \pi_E}{1 + \pi_E} \\ &= \frac{\pi_{US} - \pi_E}{1 + \pi_E} \end{aligned}$$

Now it is clear that  $\pi_{US} - \pi_E = (\pi_{US} - \pi_E)(1 + \pi_E - \pi_E)$ , so I can write this as:but

$$\frac{(1 + \pi_E)(\pi_{US} - \pi_E)}{1 + \pi_E} - \frac{\pi_E(\pi_{US} - \pi_E)}{1 + \pi_E} = (\pi_{US} - \pi_E) - \frac{\pi_E(\pi_{US} - \pi_E)}{1 + \pi_E}$$

but the last term goes to zero at normal inflation rates, so we have the result in the text. Notice that logs are much easier than this.

$\frac{s_{t+1}^e - s_t}{s_t}$ . And relative purchasing power parity tells us that  $\frac{s_{t+1}^e - s_t}{s_t} = \pi_{US}^e - \pi_E^e$ , so  $i_{US} - i_E = \pi_{US}^e - \pi_E^e$ . But this must mean that  $r_{US} = r_E$ . The key point is that under purchasing power parity all goods are tradable, so arbitrage must equate real returns.

- (c) *How does your answer to part (b) change if purchasing power parity does not hold? Explain.*

brief answer If PPP does not hold then the real exchange rate can change. In this case the expected change in the real exchange rate depends on expected movements in the nominal exchange rate and expected movements in prices levels, so  $\frac{Q_t^e - Q_{t-1}}{Q_{t-1}} = \frac{s_{t-1}^e - s_{t-1}}{s_{t-1}} - (\pi_{US}^e - \pi_E^e)$ . Using the interest parity condition, we can write this as  $\frac{Q_t^e - Q_{t-1}}{Q_{t-1}} = i_{US} - i_E - (\pi_{US}^e - \pi_E^e) = r_{US} - r_E$ . Now real interest differentials are not arbitrated away. What happens to the real interest rate differential depends on whether the increase in inflation is caused by differences in traded goods prices or non-traded goods prices. Not enough information to tell from the question what will happen to the differential, but we know it is not likely to be unchanged. Clearly, if PPP does not hold then  $\frac{s_{t-1}^e - s_{t-1}}{s_{t-1}} \neq (\pi_{US}^e - \pi_E^e)$ . So if  $\frac{s_{t-1}^e - s_{t-1}}{s_{t-1}} > (\pi_{US}^e - \pi_E^e)$  it follows that  $\frac{Q_t^e - Q_{t-1}}{Q_{t-1}}$  rises and so does the real interest differential. If  $\frac{s_{t-1}^e - s_{t-1}}{s_{t-1}} < (\pi_{US}^e - \pi_E^e)$  it follows that  $\frac{Q_t^e - Q_{t-1}}{Q_{t-1}}$  falls, and likewise so does  $r_{US} - r_E$ .

3. (30%) *Suppose our economy has a fixed exchange rate (or is on the gold standard). If the balance of payments is in surplus what happens to our international reserves (or stock of gold)? Explain. What is meant by the phrase "the balance of payments is in surplus?" How might this arise?*

brief answer If the balance of payments is in surplus this means that international reserves are increasing. The phrase means that the sum of the current account balance (CA) and the capital account balance (KO) is in surplus. Technically, an accountant might say that the balance of payments must always add up to zero if we write it as  $CA + KO + ORT$ , where ORT is official settlements (foreign purchases of dollars minus domestic purchases of foreign reserves). But typically we just write  $CA + KO = \Delta IR$ . This usage treats the change in reserves as passive, following from what happens with the current account and the capital account. Clearly, a balance of payments surplus could occur in various ways:  $CA > 0$  and  $KO > 0$  – we have positive net exports and we have positive net capital inflows,  $CA > 0$  and  $KO < 0$  but  $CA + KO > 0$  (we have a positive current account balance, but a net capital outflow as we invest in the rest of the world, as in most of the Bretton Woods period), and  $CA < 0, KO > 0$ , but  $KO > |CA|$ , a big capital inflow is larger than a small current account deficit.

- (a) *Suppose that the central bank does not engage in sterilization. What happens to the stock of money? Explain.*

brief answer If there is no sterilization then  $\Delta IR > 0$  implies that the assets of the central bank rise, so its liabilities must rise, so the monetary base rises, and so the money stock must rise. Recall that  $M = \mu MB = \mu(IR + DS)$ , where  $\mu$  is

the money multiplier and  $DS$  are domestic securities held by the central bank. So  $\Delta M = \mu \Delta IR$ .

- (b) *If the central bank engaged in sterilization what would they do? Why might they want to sterilize?*

**brief answer** Sterilization is a monetary operation to offset the effect on the money supply of the change in reserves. In this case they central bank would sell domestic securities. If  $\Delta DS = -\Delta IR$  then there is no change in the monetary base and no change in the money supply. The central bank may want to do this to prevent inflation. Otherwise the balance of payments surplus leads to more money creation and upward pressure on the price level. China is currently sterilizing inflows (mostly caused by  $KO > 0$ , in China  $CA$  is close to zero despite the large bilateral surplus with the US) to prevent upward pressure on domestic prices. Otherwise the rise in prices would reduce their competitiveness (and then they might as well just let the currency appreciate).

- (c) *Suppose the central bank adjusts the fixed exchange rate to eliminate this surplus. What would happen to the wealth of the central bank?*

**brief answer** If the central bank lets the exchange rate float the currency will appreciate. This means that the value of the reserves they have been accumulating will fall. So they will take a capital loss. Notice what this means. Countries that hold large amounts of dollar reserves stand to lose if the dollar depreciates significantly.<sup>4</sup> So they may be tempted to switch their reserves to euros, which would hasten the depreciation of the dollar. Of course, they hold back because they are afraid of losing their competitive edge in exporting to the US. But at some point they may feel that the expected capital loss will outweigh the gain in terms of net exports. If we knew when this would occur we could really make money.

## Appendix

We want to know how the required change in the real exchange rate varies with the openness of the economy.

We know that at unchanged income the trade balance improves by  $\phi \Delta q \equiv x$ . But this is not the full effect on the trade balance because income will also change. There are two effects to consider. First, we calculate how large the change in income is given  $x$ , then we calculate how the trade balance changes.

First, the change in income. This is given by

$$\Delta Y = \frac{x}{1 - a + m} \quad (1)$$

This is just the horizontal distance that the IS curve shifts when the trade balance improves by  $x$  due to the change in  $q$ .

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<sup>4</sup>At the end of August, the combined holdings of US treasury bills by China, Japan, South Korea, Singapore and Taiwan was about \$1.1 trillion. Suppose that the dollar depreciates about 20% in trade-weighted terms. The value of these holdings would fall by roughly \$200 billion. That is not a small amount!

This is not the end of the analysis, however, because when income changes the trade balance deteriorates further by  $m\Delta Y$  due to increased imports. But it also improves by  $(1 - a)\Delta Y$  due to increased savings over investment. So the total effect on the trade balance is

$$\Delta T = x - m\Delta Y + (1 - a)\Delta Y \tag{2}$$

$$= x - m \frac{x}{1 - a + m} + [1 - a] \frac{x}{1 - a + m} \tag{3}$$

$$= x \left[ 1 - \frac{m}{1 - a + m} + \frac{1 - a}{1 - a + m} \right] \tag{4}$$

It is useful to write  $1 - a \equiv b$ . Then (4) can be written as:

$$\Delta T = x \left[ 1 - \frac{m}{b + m} + \frac{b}{b + m} \right] \tag{5}$$

Now consider the case where all  $a = 1$ , that is national savings less investment is invariant to income. Then  $b = 0$ , and (5) becomes:

$$\Delta T = x \left[ 1 - \frac{m}{m} \right] = 0$$

In other words, the trade balance does not improve no matter how large is the appreciation of the real exchange rate. This is the case where the  $NS - I$  curve is horizontal. Only a change in absorption can remedy the trade balance in this case.

Now suppose that  $b > 0$  (the standard Keynesian case where  $a < 1$ ). Equation (5) can be written as:

$$\Delta T = x \left[ 1 + \frac{b - m}{b + m} \right] \tag{6}$$

We have three cases.

**Case 1** The simplest case is then if  $b = m$ . Then we have  $\Delta T = x$ , so the induced income effects offset each other, and our previous calculation is sufficient. All we need to know is the value of  $\phi$ , which depends on the elasticities of exports and imports with respect to  $q$ . The last two terms in expression (1) add up to zero.

**Case 2** If  $b > m$ , then the induced effect (of the income change) improves the trade balance. Hence the required change in  $q$  will be smaller than in case 1, since  $\frac{\Delta T}{x} > 1$ . In this case the savings effect is greater than the increase in imports. So we need less than a \$500 billion improvement due to  $\Delta q$  at unchanged income. The last two terms in expression (1) sum to a positive number, so the total effect on the trade balance is larger than  $x$ .

**Case 3** If  $b < m$ , then we have  $\frac{\Delta T}{x} < 1$ . In this case we need a larger change in the real exchange rate. In this case the increase in savings is insufficient to offset the effect of increased imports. The induced effect from the income change reduces the impact on the trade balance. In this case the last two terms in expression (1) sum to a negative number so the total effect on the trade balance is less than  $x$ .

Using (6) we can solve for the required change in  $x$ :

$$x = \Delta T \frac{1}{1 + \frac{b-m}{b+m}} = \Delta T \frac{b+m}{2b} \quad (7)$$

Now to our calculations. Again start with exports = 600, imports = 1100, and  $\eta = \eta^* = 1$ . Then we have

$$\Delta q(600 + 1100) = 500 \frac{b+m}{2b}$$

so

$$\Delta q_{open} = \frac{500}{1700} \frac{b+m}{2b} \quad (8)$$

If the economy is less open, let exports be 100 and imports be 600, then we have

$$\Delta q(100 + 600) = 500 \frac{b+m}{2b}$$

so

$$\Delta q_{closed} = \frac{500}{700} \frac{b+m}{2b} \quad (9)$$

so comparing (8) and (9) we see that  $\Delta q_{open} > \Delta q_{closed}$ , except when  $b = 0$ , in which case the required change in the real exchange rate is infinite.