

Economics 503 Fall 1999

Problem Set IV

The due date for this assignment is Tuesday, November 30, 1999 (in class).

1. Consider the standard Diamond model with population growth equal to n , and no bequests. Suppose utility is given by:

$$U = \log c_{1t} + \frac{1}{1+\rho} \log c_{2,t+1} \quad (1)$$

where ρ is the rate of time preference. Suppose that production in this economy is given by:

$$f(k) = Ak^\alpha - \delta \quad (2)$$

where δ is the rate of depreciation.

- (a) Show that given 1 savings in this model will be proportional to the wage rate, and will be independent of the rate of interest. Explain this result.
 - (b) Assuming that workers are paid their marginal products write down the expression for the equilibrium wage rate.
 - (c) Use your answer to part (b) with your answer to part (a) to derive an expression for k_{t+1} as a function of k_t and the parameters in the model. Show that this can be written as $k_{t+1} = Dk_t^\alpha$, where D is a function of the parameters of the model.
 - (d) Use your result in part (c) to solve for the steady-state capital labor ratio. Will the equilibrium be dynamically efficient? Explain.
 - (e) Provide a graphical analysis to explain whether the equilibrium in this model is stable.
2. Continue with the model in question 1. Suppose now that a social security system is established. Each agent is taxed an amount d_t when young, and receive a benefit b_t when old. A fully-funded (FF) system is defined by

$$b_t = (1 + r_t)d_{t-1} \quad (\text{FF})$$

while a pay-as-you-go system (PYG), as we have in the US is defined by:

$$b_t = (1 + n)d_{t-1}. \quad (\text{PYG})$$

- (a) Show that as long as d_t is less than planned savings, that the FF system does not affect the steady-state equilibrium.
- (b) Show that in the PYG system social security that everyone is made better off if initially $r < n$. What happens if $r > n$?
- (c) Suppose that $r > n$. What happens to the steady state value of k if d is increased? Interpret this result.
- (d) How would your answers change, if at all, if agents made operative, altruistic bequests? Explain.