Lecture Note on the Real Exchange Rate

Barry W. Ickes

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0.1 Introduction

The real exchange rate is the critical variable (along with the rate of interest) in determining the capital account. As we shall see, this is because the real exchange rate is the relative price of goods across countries. Hence, changes in the real exchange rate affect the competitiveness of traded goods.

The nominal exchange rate, S, refers to the dollar price of foreign exchange.[1] As with most variables in economics we distinguish between the nominal and *real* values. The real exchange rate measures the cost of foreign goods relative to domestic goods. It gives a measure of competitiveness, and it is a useful variable for explaining trade behavior and national income.

One of the great puzzles in international macroeconomics is why the real exchange rate is so volatile. Consider figure 1 which shows the real exchange rate since 1973. You can see that the real exchange rate is not only volatile, it does not appear to move around some equilibrium level. There are long swings. Given that this is a relative price one would suspect that such large changes would have big welfare implications on the economy. This is a second surprise – it appears that this is not the case either. These are two points to think about.

0.2 Definition

We can define the real exchange rate, Q, by

$$Q = \frac{SP^*}{P} \tag{1}$$

[1] I use an S to denote spot exchange rate.

Figure 1: Real Exchange Rate

where P^* is the price level in the foreign country. An *appreciation* of the real exchange rate indicates that the foreign price (in dollars) of a bundle of goods has risen relative to the domestic price. If the real exchange rate appreciates it means that the real value of the dollar has depreciated; that is, the purchasing power of the dollar has fallen in relative terms.

Notice that to define the real exchange rate we need to specify the price levels. If the baskets of goods in the domestic and foreign countries were the same this would be straightforward; in practice, they are not. We typically use some broad measure of the price level, such as the GDP deflator or the CPI. It should be noted that this means that P will place a relatively heavy weight on goods produced and consumed domestically, while P^* will likewise place a relatively heavier weight on goods produced in the foreign country.[2] We will soon see the importance of this.

What causes changes in Q? Two specific causes are worth discussing here.

1. *A change in world relative demand for US goods.* Suppose that preferences shifted so that total world spending on US goods increased. This could be due to shifts in private demand towards US goods, or an increase in US government spending which is concentrated on US goods. At current exchange rates this would cause an excess demand for US goods. To restore equilibrium the relative price of US goods

[2] This is especially true because of non-traded goods, which we shall discuss shortly.

must rise relative to foreign goods; hence, Q must fall, and the dollar has appreciated in real terms. In other words, the purchasing power of the dollar has increased relative to foreign goods.

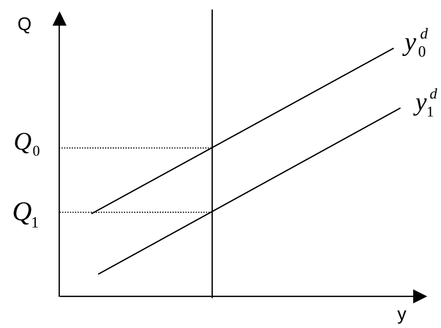

Figure 2: An Increase in the Demand for Domestic Goods

2. *A change in relative output supply.* Suppose that there is a relative technological shock that increases the efficiency of US output relative to foreign output. With given stocks of capital and labor US output rises. Hence, at unchanged world demand there is an excess supply of US output. Why? This positive supply shock raises US income (wealth), but not all of the increase in income is spent on domestic goods. Some will be spent on foreign goods. Hence, the increase in the demand for US goods will be less than the supply. To restore equilibrium the relative price of US goods must fall; in other words, Q must rise, and the dollar must fall in real terms. This real depreciation of the dollar (or real appreciation of the foreign currency, say the DM) means that the purchasing power of the foreign currency has increased. Thus relative productivity growth causes the real exchange rate to appreciate and the real value of the currency to depreciate.

We will return to the topic of real exchange rate movements after we take a detour to discuss exchange rate determination when the real exchange rate is constant.

Why is the real exchange rate so important for thinking about the current account? Because it is the relative price of foreign goods in terms of domestic

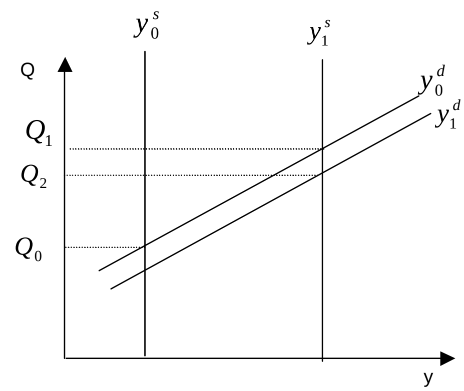

Figure 3: A change in relative supply

goods, changes in this variable will impact net exports, and hence, the current account. If a current account deficit is to be reversed an appreciation of the real exchange rate may be one of the mechanisms of adjustment.

0.2.1 A Special Case: Purchasing Power Parity

An interesting case to consider is the special case where the real exchange rate is constant over time. Suppose that the basket of goods that were produced in the US and Germany were identical, and that all goods were tradeable. In that case, net of transportation costs we would have the law of one price: arbitrage would insure that the dollar prices of the various goods would be identical across countries. This yields a theory of exchange rate determination known as PPP.

Notice that we could use the definition of the real exchange rate to write:

$$S_t = \frac{Q_t P}{P*} \tag{2}$$

Now suppose that the real exchange rate is constant over time; hence, $Q_t = \overline{Q}$ for all time. Then

$$S_t = \frac{\overline{Q} P}{P*} \tag{3}$$

and it follows that any changes in national price levels results in a movement of the exchange rate. PPP thus determines the exchange rate by the movements in relative price levels. If US inflation is higher than foreign inflation the exchange rate will appreciate and the dollar will depreciate relative to the foreign currency. It will take more dollars to purchase a DM. This is intuitive: the nominal exchange rate is the relative price of currencies, and inflation is the measure of the decrease in purchasing power of a currency. If the dollar is losing purchasing power faster than a DM, then the DM should gain in value relative to the dollar.

This can be seen more clearly, perhaps, by taking logs of both sides of (3):

$$s_t = \overline{q} + p_t - p_t^* \tag{4}$$

where we have used lower-case letters to refer to the log of a variable.[3] Now suppose we take first differences of (4), i.e., $\Delta s_t \equiv s_t - s_{t-1}$:

$$\Delta s_t = \Delta p_t - \Delta p_t^*. \tag{5}$$

Expression (5) says that the percentage change in the nominal exchange rate is equal to the difference between the inflation rates in the domestic and the foreign country.[4] When price levels are changing very rapidly these movements can dwarf all other factors, and then PPP provides a rather effective theory of exchange rate movements.

A simple example of this theory is provided by the Big Mac index. The Big Mac is essentially the same good in every country. Hence, we can compare the dollar price of Big Mac's across countries. Where the currency appears over-valued we should expect the exchange rate to appreciate, and vice versa. This does surprisingly well. See figure.

There are several reasons why PPP does not hold in the short run. Notice, that PPP is a theory of exchange rate determination based on goods flows. It is tied to trade (it is not the only theory of this kind), and it ignores capital flows. Exchange rates can also fluctuate because of expectations of future changes, though even these must be based on something. We have talked about current accounts. Of course, the need to finance deficits can lead to different rates of inflation, and so back to PPP. So it is not trivial to dismiss it. Here are some important issues.

[3] If you detest taking logs see equation 20 where we derived the same expression without taking logs.

[4] Recall that $\frac{X_t - X_{t-1}}{X_{t-1}} = \frac{X_t}{X_{t-1}} - 1$ is the percentage change in X, and thus $1+g = \frac{X_t}{X_{t-1}}$, where x is the percentage growth rate. Now if we take logs of this expression, for small g, it follws that $log(1+g) \approx g \approx x_t - x_{t-1} \equiv \Delta x_t$.

- First, tariffs and transportation costs create a band in which prices can fluctuate before arbitrage becomes profitable.

- Second, permanent shifts in the terms of trade can cause Q to change, if countries differ in the composition of output. An oil shock (positive) will have a different effect on an energy producer and a producer of energy-intensive products. The latter country will experience a relative decline in the world demand for its goods, so its currency will experience a real depreciation.

- Third, if prices are sticky in the short run the law of one price, by definition, does not hold. Then it follows that movements in nominal exchange rates will also affect the real exchange rate. This may hold in the short run, but over longer periods of time prices do adjust and PPP is more likely to hold.

- Fourth, the presence of *non-traded* goods, is probably the most important factor. Think of haircuts versus wheat. Even if traded goods are identical across countries and obey the law of one price, non-traded goods do not. Shifts in the relative price of traded and non-traded goods can cause PPP to fail. This is rather easy to see.

Let us write the price index of the domestic country as $P = P_n^{\alpha} P_t^{1-\alpha}$, where P_t is the price of traded goods, and α is the share of non-traded goods in the domestic price index. Now we can write the real exchange rate as:

$$\begin{aligned} Q &= S\left[\frac{P_n^{*\alpha^*} P_t^{*(1-\alpha^*)}}{P_n^{\alpha} P_t^{1-\alpha}}\right] = S\left[\frac{\left(\frac{P_n^*}{P_t^*}\right)^{\alpha^*} P_t^*}{\left(\frac{P_n}{P_t}\right)^{\alpha} P_t}\right] \\ &= S\left(\frac{P_t^*}{P_t}\right)\left[\frac{\left(\frac{P_n^*}{P_t^*}\right)^{\alpha^*}}{\left(\frac{P_n}{P_t}\right)^{\alpha}}\right] \end{aligned}$$

but if we assume that PPP holds for traded goods, it follows that $S\left(\frac{P_t^*}{P_t}\right) = 1$, so

$$Q = \left[\frac{\left(\frac{P_n^*}{P_t^*}\right)^{\alpha^*}}{\left(\frac{P_n}{P_t}\right)^{\alpha}}\right]. \tag{6}$$

Expression (6) tells us that the real exchange rate will change if the *relative* price of non-traded goods changes in either the domestic or foreign country.

Notice that if take logs of both sides of (6), and use lower-case to represent the log of a variable, we obtain:

$$q = \alpha^*(p_n^* - p_t^*) - \alpha(p_n - p_t)$$

taking first differences, we obtain:

$$\Delta q = \alpha^*(\Delta p_n^* - \Delta p_t^*) - \alpha(\Delta p_n - \Delta p_t) \tag{7}$$

which says that the real exchange rate will depreciate if the relative price of non-traded goods (i.e., relative to traded goods) rises in the domestic country or decreases in the foreign country.[5]

There is good reason to think that such changes do occur. The Balassa-Samuelson effect focuses on the impact of differential economic growth. It is argued that economic growth is associated with increased productivity in *traded* goods, so that they fall relative to the price of *non-traded* goods. You can think of traded goods as more tangible than non-traded goods (like haircuts). In countries that grow rapidly (or liberalize for that matter) non-traded goods will rise relative to traded goods. If this happens more rapidly in the domestic economy than in the rest of the world then q would fall.

This could also happen if non-traded goods are superior in consumer's demand functions. Either way, this relative price change causes the real exchange rate to decrease; in other words, the real value of the domestic country appreciates. This is, of course, what happened in Japan as rapid growth lead to very rapid increases in the price of non-traded goods (such as golf club memberships).

The fact that q is lower in countries that grow faster may also explain why the price level tends to be higher in richer countries when measured in common currency units. Americans often wonder how people in LDCs can live on incomes of $500 a year. Of course, there is real poverty, but it is also the case that because of non-traded goods, conversion at exchange rates gives an incorrect impression. That is, the differences in nominal incomes do not

[5]Notice that if $\alpha = \alpha^*$, this can be simplifed to:

$$\Delta q = \alpha(\Delta p_t - \Delta p_n) - \alpha(\Delta p_t^* - \Delta p_n^*).$$

measure the true differences in purchasing power. This is because purchasing power of a currency differs depending on the shares of traded and non-traded goods. To see this, recall that from the definition of the real exchange rate, $Q_t = \frac{S_t P_t^*}{P_t}$, we can write:

$$q_t = s_t + p_t^* - p_t$$

hence,

$$p_t = s_t + p_t^* - q_t \tag{8}$$

which implies that countries with lower q will have higher price levels compared with prices elsewhere, since the foreign price level measured in units of domestic currency is just $s_t + p_t^*$. If productivity growth is rapid in the US relative to the foreign country, our price level will be higher, when measured in common currency units.

One way to think about this is simply that PPP values exchange rates according to the relative price of traded goods. But in LDC's the price of non-traded goods is lower. When these are included, the price level in the advanced country is higher. That is essentially what is implied by 8.

0.2.2 The Role of Productivity Growth

Here is a straightforward way to see why a country that has rapid growth in productivity experiences real exchange rate appreciation. The key notion is what happens to wages.

Start again with the definition of the price level as $P = P_n^{\alpha} P_t^{1-\alpha}$, and use asterisks for the foreign country. The law of one price implies that for tradable goods we have

$$P_t = eP_t^* \tag{9}$$

Now profit maximization means that wages equal marginal products. So we will have

$$\frac{w_t}{P_t} = MPL_t \text{ and } \frac{w_t^*}{P_t^*} = MPL_t^* \tag{10a}$$

or $P_t = \frac{MPL_t}{w_t}$. Then it follows that

$$\frac{MPL_t}{w_t} = e\frac{MPL_t^*}{w_t^*}$$

or

$$\frac{ew_t}{w_t^*} = \frac{MPL_t}{MPL_t^*} \tag{11}$$

Notice the implication of (11): the ratio of dollar wages in the tradeable goods sector is equal to the ratio of marginal products in traded goods. If productivity rises in tradable goods in country a so will its wage rate in tradeables.

Now we connect wages in tradeables and non-tradeables in a given country. Labor market equilibrium requires that wages equalize. As a first approximation we will let $w_t = w_n = w$, and $w_t^* = w_n^* = w^*$. But profit maximization must imply that conditions like (10a) hold for non tradables:

$$\frac{w_n}{P_n} = MPL_n \text{ and } \frac{w_n^*}{P_n^*} = MPL_n^*$$

or

$$w_n = P_n MPL_n \text{ and } w_n^* = P_n^* MPL_n^*. \tag{12}$$

Our final step is to consider productivity in non-tradables. Since our concern is with productivity growth in tradables it is not a bad assumption to assume that non-tradable productivity is equal across sectors. Not much capital goes into a haircut. So we let $MPL_n = MPL_n^*$. Nothing would be afffected if we let $MPL_n = \gamma MPL_n^*$ for example, as long as γ was constant.

Now we just put together the pieces. Start with the definition of the real exchange rate

$$\begin{aligned} Q &= \frac{eP^*}{P} = e\frac{P_n^{*\alpha} P_t^{*1-\alpha}}{P_n^{\alpha} P_t^{1-\alpha}} = \left(\frac{eP_n^*}{P_n}\right)^{\alpha} \left(\frac{eP_t^*}{P_t}\right)^{1-\alpha} \\ &= \left(\frac{eP_n^*}{P_n}\right)^{\alpha} \end{aligned}$$

since from (9) we know that $\left(\frac{eP_t^*}{P_t}\right)^{1-\alpha} = 1$. Now use expression (12) and the fact that $MPL_n = MPL_n^*$ to substitute for the price of non-tradeables:

$$\begin{aligned} Q &= \left(\frac{eP_n^*}{P_n}\right)^{\alpha} = \left(\frac{ew_n^*}{w_n}\right)^{\alpha} = \left(\frac{ew^*}{w}\right)^{\alpha} \\ &= \left(\frac{eP_t^* MPL_t^*}{P_t MPL_t}\right)^{\alpha} = \left(\frac{MPL_t^*}{MPL_t}\right)^{\alpha} \end{aligned} \tag{13}$$

so we have shown that the real exchange rate depends on the ratio of marginal products of labor in tradeables. It is apparent then that if the marginal product of labor in the tradeable goods sector rises in the foreign country relative to the home country the real exchange rate must appreciate. To see this formally, just take logs of (13) and differentiate with respect to time to get an expression for the growth rate of the real exchange rate (hats denote growth rates):

$$\widehat{Q} = \alpha \left[\widehat{MPL}_t^* - \widehat{MPL}_t \right]. \tag{14}$$

What does (14) imply? First, if all goods were tradeable, $\alpha = 0$, and thus the real exchange rate is constant. The higher the share of non-tradeables the greater the impact of differential productivity growth on the change in Q.

Rising Yen The Balassa-Samuelson effect may also help explain the rising yen. In nominal terms the yen has strengthened greatly since WW2. Between 1950 and 1999 the dollar lost two-thirds of its value against the yen. Notice that much of this happened when there was a fixed exchange rate between the dollar and yen. What it reflects is higher Japanese inflation prior to 1973 than in the US. But subsequent to that US inflation was higher than in Japan. Yet, the movements in the exchange rate cannot be due to differences in inflation alone, however, as US inflation has not been that much higher than Japanese (though it has been and continues).

From the data we see that in real terms the dollar has depreciated against the yen for more than forty years. Why? Differential productivity growth in traded and non-traded goods. The relative price of non-traded goods in Japan has increased much more than in the US. After WW2 non-tradables in Japan were very c heap because the economy was still in recovery. As Japan recovered productivity increased in traded goods. The overall consumption basket must have been very cheap in that period. As the economy recovered the relative price of non-traded goods increased. This follows as productivity in the traded goods sector rises to world levels. Why? Because wages in the non-traded goods sector must rise as wages increase in the traded goods sector. It is harder to improve productivity in non-traded goods sectors. Think of golf club memberships.

The same result occurs in transition economies. Their traded goods sectors were very inefficient at the start of transition. As their economies improve the relative price of tradeables rises. This raises average wages in the economy and their price levels rise relative to foreign prices – their currencies appreciate in real terms. Of course, in these economies the appreciation is also due to recovery from the depreciated exchange rates that resulted from the initial collapse of their currencies and capital flight.

0.2.3 Interest Differentials and the Real Exchange Rate

If PPP holds, then interest differentials are a function of differences in expected inflation. The law of one price suggests that the spot exchange rate is determined by relative price levels:

$$e_t = \frac{P_{US}}{P_E} \tag{15}$$

Expression (15) is a theory of exchange rate determination – *purchasing power parity* – based on the assumption that all goods are tradeable.[6] Hence, it assumes that real exchange rates are constant. It is not a bad assumption for the long run, but it may be problematic for the short run. If each country produced one and the same good, and if transport costs and national prejudices did not exist, then arbitrage would clearly bring about (15). Of course countries produce many goods, and not all are tradeable. It is nonetheless worthwhile to see its implications.

From (15) we can write:

$$\frac{e_t}{e_{t-1}} = \frac{\frac{P_{US,t}}{P_{E,t}}}{\frac{P_{US,t-1}}{P_{E,t-1}}} = \frac{\frac{P_{US,t}}{P_{US,t-1}}}{\frac{P_{E,t}}{P_{E,t-1}}} \tag{16}$$

Now define inflation as $\pi_t = \frac{P_{US,t}}{P_{US,t-1}} - 1$. So we can write (16) as:

$$\frac{e_t - e_{t-1}}{e_{t-1}} = \frac{1+\pi_{us}}{1+\pi_E} - 1 = \frac{1+\pi_{us}}{1+\pi_E} - \frac{1+\pi_E}{1+\pi_E} \tag{17}$$

$$= \frac{\pi_{US} - \pi_E}{1+\pi_E} \tag{18}$$

Now it is clear that $\pi_{US} - \pi_E = (\pi_{US} - \pi_E)(1 + \pi_E - \pi_E)$, so I can write (18) as:

$$\frac{(1+\pi_E)(\pi_{US} - \pi_E)}{1+\pi_E} - \frac{\pi_E\,(\pi_{US} - \pi_E)}{1+\pi_E} = (\pi_{US} - \pi_E) - \frac{\pi_E\,(\pi_{US} - \pi_E)}{1+\pi_E} \tag{19}$$

[6] Suppose that the basket of goods that were produced in the US and Germany were identical, and that all goods were tradeable. In that case, net of transportation costs we would have the law of one price: arbitrage would insure that the dollar prices of the various goods would be identical across countries. This yields a theory of exchange rate determination known as PPP.

But if inflation rates are rather low the difference between them is likely to be low, and the product of this difference and the inflation rate is likely to be even lower. Hence, for low inflation rates the last term on the right hand side of 19 $\rightarrow 0$, which means that we have the approximation:

$$\frac{e_t - e_{t-1}}{e_{t-1}} = \pi_{US} - \pi_E \tag{20}$$

which is called *relative purchasing power parity.*

Expression (20) says that the percentage change in the nominal exchange rate is equal to the difference between the inflation rates in the domestic and the foreign country.[7] When price levels are changing very rapidly these movements can dwarf all other factors, and then PPP provides a rather effective theory of exchange rate movements. A great advantage of this expression is that it holds even if absolute PPP does not.

Now recall that from UIPC we have:

$$\frac{e^e_{t+1} - e_t}{e_t} = i_{US} - i_E \tag{21}$$

So if expected inflation differences correspond with actual inflation differences, it follows that the interest differential will be equal to the difference in expected inflation rates. Moreover, as market participants understand expression (20) it follows that expected inflation will be equal to the growth rate of the exchange rate. Hence,

$$i_{US} - i_E = \pi^e_{US} - \pi^e_E \tag{22}$$

where π^e_{US} is the expected US inflation rate. Thus expression (22) says that the interest differential will be equal to the difference in expected inflation rates.

Notice what expression (22) implies. If agents expect higher US inflation relative to Europe it follows that US interest rates must rise relative to European rates. Hence, according to expression (22) the real return on US assets relative to Europe will be unchanged. This is called the *Fisher effect.* The *Fisher effect* is usually written as $r = i - \pi^e$. Movements in expected

[7]Recall that $\frac{X_t - X_{t-1}}{X_{t-1}} = \frac{X_t}{X_{t-1}} - 1$ is the percentage change in X, and thus $1+g = \frac{X_t}{X_{t-1}}$, where x is the percentage growth rate. Now if we take logs of this expression, for small g, it follws that $log(1+g) \approx g \approx x_t - x_{t-1} \equiv \Delta x_t$.

inflation leave real interest rates unchanged. But this is also what is implied by expression (22).

Suppose then that money growth in the US rises relative to Europe. In the long run we would expect that inflation would rise by an equal amount. And so would expected inflation. Hence we would expect the interest rate in the US to rise relative to Europe. This also implies that the exchange rate must appreciate at the same rate as the interest differential. In the long run output and real returns are unchanged. The only effect of the rise in money growth is on the nominal quantities. Of course, we know in the short run there will be impacts. But we can also see that in the long run only nominal quantities are effected.

A simple example of this theory is provided by the Big Mac index. The Big Mac is essentially the same good in every country. Hence, we can compare the dollar price of Big Mac's across countries. Where the currency appears over-valued we should expect the exchange rate to appreciate, and vice versa. This does surprisingly well though it is not perfect. Notice that even with the Big Mac, however, price differences persist. Not even all the Big Mac costs are really tradeable.

The assumption of PPP is equivalent to assuming that the real exchange rate is constant. Yet, we know it is not. So how is the theory modified? Recall that the real exchange rate is defined as $Q_t = \frac{e_t P_t^E}{P_t^{US}}$. We are interested in an expression for the *expected* growth rate of the real exchange rate, $\frac{Q_t^e - Q_{t-1}}{Q_{t-1}}$. Suppose that inflation was expected to equal in the US and euroland. Then clearly we would have $\frac{Q_t^e - Q_{t-1}}{Q_{t-1}} = \frac{e_t^e - e_{t-1}}{e_{t-1}}$. Of course expected inflation rates are not equal, however, so how is the expression altered? Suppose that the exchange rate was not expected to change. Then clearly we would have $\frac{Q_t^e - Q_{t-1}}{Q_{t-1}} = \pi_E^e - \pi_{US}^e$. If US inflation is higher than in euroland the real exchange rate depreciates. Put these two factors together and it is clear that:

$$\frac{Q_t^e - Q_{t-1}}{Q_{t-1}} = \frac{e_t^e - e_{t-1}}{e_{t-1}} - (\pi_{US}^e - \pi_E^e). \tag{23}$$

The expression is intuitive.[8]

Now recall the interest parity condition, $\frac{e_t^e - e_{t-1}}{e_{t-1}} = i_{US} - i_E$. Using this to

[8] To prove this take logs of the expression for the real exchange rate expression:

$$\log Q_t = \log e_t + \log P_t^E - \log P_t^{US}$$

replace the expected rate of depreciation in (23), we obtain:

$$i_{US} - i_E = \frac{Q_t^e - Q_{t-1}}{Q_{t-1}} + (\pi_{US}^e - \pi_E^e) \tag{24}$$

which implies that interest differentials depend on expected movements in the real exchange rate in addition to differences in expected inflation. When the real exchange rate is not expected to change we have the same expression as before. But in general, nominal interest differentials are explained by movements in the real exchange rate as well. Of course you can think of the latter as capturing all the reasons for exchange rate movements other than differential price levels.

This now lets us derive an expression for *real interest parity.* While nominal returns are those that are the actual components of exchanges, it is real returns, or rather *expected* real returns that govern decisionmaking. That is, you may earn a nominal interest rate of 10% from an asset, but whether you choose to hold it or not depends on how the real return relates to the opportunity cost of the funds. So an investor thinking of where to hold her wealth must consider the expected real returns.

We know that nominal interest differentials are related to expected changes in the real exchange rate and expected inflation. We obtain the real interest parity condition by first using the Fisher equation, $i_t = r_t + \pi^e$. It follows that the expected real rate of interest in the US, $r_{US,t}^e = i_{US,t} - \pi_{US,t}^e$, and likewise for Euroland, $r_{E,t}^e = i_{E,t} - \pi_{E,t}^e$. Using these in expression (24) yields the real interest parity condition:

$$r_{US,t}^e - r_{E,t}^e = \frac{Q_t^e - Q_{t-1}}{Q_{t-1}} \tag{25}$$

which says that expected real interest rate differentials are equal to expected changes in the real exchange rate.

Why should (25) hold? Suppose that people expect the real exchange rate to appreciate, say because Euroland productivity growth in tradeables

Now differentiate both sides with respect to time and we obtain:

$$\left(\frac{1}{Q_t}\right)\frac{dQ_t}{dt} = \left(\frac{1}{e_t}\right)\frac{de_t}{dt} + \left(\frac{1}{P_t^E}\right)\frac{dP_t^E}{dt} - \left(\frac{1}{P_t^{US}}\right)\frac{dP_t^{US}}{dt}.$$

But these expressions are just the (continuous time) growth rates of the real and nominal exchange rate and of the price levels in euroland and the US, respectively. So as the period shrinks, we obtain the expression in the text.

is expected to be higher than in Euroland non-tradeables, and also higher than in the US. Thus people expect the real value of the dollar to depreciate relative to the euro. To compensate for this the real return on dollar assets must exceed those of Euroland assets.

Does this mean that there are profit opportunities that are not being arbitraged away? No. The differences in the real rates do not reflect different returns on the *same asset*. It reflects different returns on two bundles of goods. The absence of arbitrage opportunities is guaranteed by interest parity, since any investor that compares relative returns has a unique consumption basket. When I compare the rate of return on holding dollars or euros, the real return is computed by subtracting my expected rate of inflation, whatever consumption basket is relevant for me. But the expected real differential on the left-hand side of (25) is comparing two expected inflation rates that reflect two *different* consumption baskets. Notice that if all agents were identical PPP would hold and we would not have real interest differentials. But because people in different countries consume different baskets of goods, there is no way for them to arbitrage away any difference.

Why is this interesting? Recall when we spoke of the Feldstein-Horioka puzzle we commented that a direct way to test for capital market integration would be to look at whether excess returns were arbitraged away. So that would suggest looking at real interest differentials in different countries. If capital markets are integrated these should go to zero. But as we have just seen, this is only true if PPP holds. If it does not hold, then real interest differentials should be expected to persist. Suppose, for example, that US savings was so low that people expected the real exchange rate to appreciate in the future.[9] Then real returns on US assets would have to exceed those in the rest of the world, according to (25).

[9] Why do they expect Q to appreciate? We have to pay back the debts we have incurred, so current account deficits must become surpluses. So the relative price of our goods must fall relative to the rest of the world to allow us to export more and import less.